REPORT RESUMES

ED 003 818 24

DEDUCTIVE REASONING IN ADOLESCENCE--CRITICAL THINKING READINESS IN GRADES 1-12, PHASE 1.

BY- ENNIS, ROBERT H. PAULUS, DIETER
STATE UNIV. OF N.Y., ITHACA
REPORT NUMBER CRP-1680 PUB DATE 65
EDRS PRICE MF-$0.45 HC-$12.56 314P.

DESCRIPTORS- *CRITICAL THINKING, THOUGHT PROCESSES, *LOGICAL THINKING, COGNITIVE PROCESSES, ADOLESCENTS, *COGNITIVE DEVELOPMENT, ITHACA, NEW YORK

THIS STUDY DEALT WITH THE CONCEPT OF DEDUCTIVE LOGIC AND ITS USE BY ADOLESCENTS, AGES 10-18. TWO EMPIRICAL QUESTIONS OF CONCERN IN THIS STUDY WERE (1) THAT OF READINESS FOR MASTERY OF LOGIC AND (2) THAT OF THE NATURAL-CULTURAL DEVELOPMENT OF MASTERY OF LOGIC. TWO DEDUCTION TESTS WERE DEVELOPED, ONE FOR EACH OF THE TYPES OF LOGIC TREATED. THEY WERE THE CORNELL CONDITIONAL-REASONING TEST, FORM X, AND THE CORNELL CLASS-REASONING TEST, FORM X. THE TESTING OF 803 SUBJECTS LED TO (1) THE SPECIFICATION OF A SET OF PRINCIPLES OF TWO MAJOR TYPES OF LOGIC, CONDITIONAL LOGIC AND CLASS LOGIC, (2) TWO LOGIC TESTS, (3) A THEORY OF OPERATIONAL DEFINITIONS, AND (4) AN ANALYSIS OF PIAGET'S CONCEPTION OF LOGIC. (GD)

U. S. DEPARTMENT OF HEALTH, EDUCATION AND WELFARE
Office of Education
This document has been reproduced exactly as received from the person or organization originating it. Points of view or opinions stated do not necessarily represent official Office of Education position or policy.

CRITICAL THINKING READINESS IN GRADES 1-12

(PHASE 1: DEDUCTIVE REASONING IN ADOLESCENCE)

COOPERATIVE RESEARCH PROJECT NO. OE 1680

Robert H. Ennis, Principal Investigator
and
Dieter Paulus

New York State College of Agriculture at Cornell University and
the School of Education, Cornell University
Ithaca, N. Y.

1965

The research reported herein was supported by the Cooperative Research Program of the Office of Education, U. S. Department of Health, Education, and Welfare.

ED 003818

ACKNOWLEDGEMENTS

We are indebted to the school systems of Ithaca, Lansing, and Newfield, New York, for their help and cooperation in this research. People from these systems who were particularly helpful are Mr. Leonard Backer, Miss Dorothy Balbirnie, Mr. William Beischer, Mrs. Virginia Bristow, Mr. Raymond Buckley, Dr. Leonard Buyse, Mrs. Rhoda Curtis, Mr. Warren Currier, Miss Marion Davenport, Dr. John Graves, Mrs. Florence Harley, Mr. Douglas Hart, Mr. Robert Holgate Mr. Thornton Jesdale, Mr. Robert Kellog, Dr. James Mason, Mrs. Martha Nelson, Mrs. Margaret Paetow, Mrs. Theresa Parakh, Miss Natalie Payst, Mr. Curtis Rohm,Mr. Philip Skeps, and Mrs. Ruth York.

The help given by the following faculty members at Cornell University is very much appreciated: Professors Max Black, John Canfield, Richard Darlington, Keith Donnellan, Mauritz Johnson, Jason Millman, and Frederick Stutz.

Project staff members, who gave far more than required of them, are Professor William Gardiner, Mr. John Guzzetta, Mrs. Sharon Lewton, Mrs. Carmen Mix, Mr. Richard Morrow, Professor Lucille Ringel and Miss Margaret Smith.

Our debt to the Cooperative Research Branch of the U.S. Office of Education is great. From this group we received support and encouragement without the limitations and specifications that are sometimes found to accompany Federal grants. The kind of research for which support was available was not specified in advance, and changes made necessary by discoveries as the research proceeded were accepted without comment.

RHE and DP

TABLE OF CONTENTS

		Page
Acknowledgements		ii
List of Tables		vii
List of Graphs		x
Chapter No.		
I	Introduction	I-1
II	The Subject Matter: Logic	II-1
	A. An Analysis of Critical Thinking	-1
	B. Types of Deductive Logic	-3
	1. Sentence Logic	-4
	2. Class Logic	-5
	3. Ordinal Logic	-6
	4. Other Types of Logic	-6
	C. An Elaboration of The Types of Logic Investigated In This Study.	-7
	D. A Comparison with Piaget's Logic	-15
	1. Points of Correspondence	-16
	2. Points of Noncorrespondence	-18
	Summary	-26
	E. Chapter Summary	-27
III	Basic Data on Subjects	III-1
	A. The Groups with Which We Worked	-1
	1. The "LDT's"	-1
	2. The "LNDT-1's"	-3
	3. The LNDT-2's	-3
	B. The Sources of Our Data	-4
	1. Readily Obtainable Information	-4
	2. Less Readily Obtainable Information	-5
	a. Socio-Economic Status	-5
	b. The Cornell Deduction Tests	-7
	C. The Figures	-7
IV	The Cornell Deduction Tests	IV-1
	A. The Structure of the Tests	-2
	1. The Logical Form of the Items	-3
	2. Content Components	-8
	a. Concrete Familiar	-8
	b. Symbolic	-8
	c. Unfamiliar	-8
	d. Suggestive	-9
	3. Item Format	-11
	4. Item Arrangement	-13
	B. Operational Definitions of 'Mastery of X Principle'	-14
	C. Reliability	-19
	D. Validity	-25
	1. Content Validity	-25

Chapter No. Page

IV
    2. Construct Validity -26
        a. Correlations With Familiar Measures -26
        b. Item Analysis -29
        c. Making Sense in the Context of the Study -33
  E. Suggested Further Development -33
  F. Chapter Summary -36

V The Natural-Cultural Development of Knowledge of Logic

  A. The Literature V-1
    1. Piaget's Formal Operations -1
        a. The Real Versus the Possible -2
        b. Kinds of Formal Thought -6
        c. Stages -6
        d. Questions Posed by Piaget's Work -8
    2. Other Studies -10
        a. Is there a development of logical ability as children grow older? -10
        b. Does the development come in stages? -10
        c. Is conditional logic mastered by age 11-12? -11
        d. Is class logic mastered by age 11-12? -11
        e. Is the truth-validity characteristic achieved by age 11-12? -11
        f. Within each type of logic, is there a development of one sort of thing before another and are some things more difficult than others at a given level? If so what is the nature of this differential development? -12
        Summary -13
  B. The Results -14
    1. Is there a development of logical ability as children grow older? -14
    2. Does the development come in stages? -17
    3. Is conditional logic mastered by age 11-12? -22
    4. Is class logic mastered by age 11-12? -25
    5. Is the truth-validity characteristic achieved by age 11-12? -28
    6. Within each type of logic, is there a development of one sort of thing before another? If so, what is the nature of this differential development? -29
        a. Differential Development of Knowledge of Principles of logic. -29
            1. The Fallacies -29
            2. The Converse -32
            3. The Contrapositive -33
            4. Transitivity -35
            5. The Comparability of the Two Types of Logic. -36

Chapter No. Page

V
b. Differential Development of the Three Components: Concrete Familiar, Symbolic, and Suggestive. -37
C. Chapter Summary -41
1. The Literature -41
2. Findings -42
3. Needed Research -43

VI The Development of Readiness to Master Logic

A. What Is Readiness? VI-1
1. Capacity and Willingness -1
2. Dispositional Terms -2
3. A Double Disposition -3
4. Elusiveness of Readiness -3
5. A Substitute for the Identical Person -4
6. Suitable Conditions -6
7. Summary -8
B. The Literature -9
C. Procedures -10
1. Assignment of Grade Levels to Class and Conditional Logic; The Pre-Test. -10
2. Total Teaching Effort -10
3. The Actual Teaching -11
4. The Post-Test -14
D. Results -14
1. General Qualifications -15
2. What Did Our Subjects Learn? -18
a. Conditional Reasoning -18
1. Total Scores -18
2. Component Scores -19
3. Item Group Scores -21
4. Summary and Possible Import for Teaching -24
5. Learning Words vs. Learning Logic. -25
b. Class Reasoning -26
1. Total Scores -27
2. Component Scores -27
3. Item Group Scores -28
4. Summary -29
c. Possible Explanations of the Difference Between Conditional and Class Reasoning. -29
3. Can We State What Others Are Ready To Master? -30
a. An Example of the Use of a Multiple Regression Equation for Making a Prediction. -31
b. Making Use of the Generated Predictions -32
c. Summary and Overview -37
E. Further Research -37
F. Chapter Summary -38

Chapter No. Page

VII The Past and Future of the Project
A. The Readiness Study: The Past. VII-1
B. The Tests -4
C. The Readiness Study: The Future -6

VIII Summary
A. Introduction: The Statement of the Problems VIII-1
B. The Nature of the Subject-Matter: Logic. -2
1. Two Types of Logic -3
2. Primciples of Logic -5
C. Basic Data on Subjects -8
D. The Cornell Deduction Tests -9
1. Reliability -11
2. Validity -11
E. The Natural Cultural Development of Knowledge of logic. -15
F. The Development of Readiness to Master Logic -18
1. What Our Subjects Were Ready to Master -19
2. That Which Fairly Comparable Students Are Ready to Master -21
G. Needed Further Research -23
H. Overview -25

References R-1
Cited References R-1
General References R-3

Appendix

A. Tables A1-14 and Graphs A1-4

B. The Cornell Deduction Tests
"The Cornell Conditional Reasoning Test, Form X"
"The Cornell Class Reasoning Test, Form X"
Trial Answer Sheet for Lower Elementary Students

C. Samples of Teaching Materials
Class Reasoning Exercises Used in 10th and 12th Grades, Consisting of Three Used Early in the Instructional Period and OneUsed at the End. C-1

Conditional Reasoning Exercises Used in 11th Grade, Consisting of Three Used Early in the Instructional Period and One Used at the End. C-8

A Class Reasoning Exercise Used Toward the End of the Instructional Period in 4th and 6th Grades. C-15

A Conditional Reasoning Exercise Used Toward the End of the Instructional Period in 5th Grade. C-17

LIST OF TABLES

~~Chapter~~		Page
II-1	Illustrated Basic Principles of Conditional Logic	II-10,11.
II-2	Illustrated Basic Principles of Class Logic	II-12, 13.
III-1	Percentage Occupational Make-Up of City, New York State and the United States.	III-2
III-3	Basic Information on Our Subjects	III-8, 9.
IV-1	Logical Form of and Answers to Items in "The Cornell Conditional Reasoning Test, Form X"	IV-4, 5.
IV-2	Logical Form of and Answers to Items in "The Cornell Class-Reasoning Test, Form X"	IV-6, 7.
IV-3	Test-Retest Reliability Estimates on the Conditional Reasoning Test.	IV-20
IV-4	Test-Retest Reliability Estimates on the Class Reasoning Test.	IV-21
IV-5	Standard Errors of Item Group Scores-Conditional Reasoning	IV-22
IV-6	Standard Errors of Item Group Scores-Class Reasoning	IV-23
IV-7	Correlations Between Total Scores on the Cornell Deduction Test and Certain Familiar Variables: Chronological Age, I.Q., Socio-Economic Status, and Sex.	IV-28
IV-8	Mean Difficulty and Discrimination Indices For The Class Reasoning Test, Based Upon Pre-Test of LDT's, LNDT-1's, and LNDT-2's Combined.	IV-31
V-1	Mean Chronological Age; I.Q.; Estimated Mental Age; and Total Component, and Item Group Conditional and Class Reasoning Pre-Test Scores; by Grade for all Subjects Grouped Together.	V-16
V-2	Percent Meeting the Sufficient Condition and the Percent Failing to Meet the Necessary Condition for Mastery of Each Principle at Each Grade Level on the Conditional Reasoning Test.	V-18

~~Chapter~~		Page
V-3	Percent Meeting the Sufficient Condition and the Percent Failing to Meet the Necessary Condition for Mastery of Each Principle at Each Grade Level on the Class Reasoning Test.	V-19
V-4	Difference Between Lowest Grade and Highest Grade Mean Item Group Difficulty Indices; Item Group Answers.	V-31
V-5	Comparisons of Mean Difficulty Indices of Three Components: Concrete Familiar, Symbolic, and Suggestive.	V-38
VI-1	Rough ~~Teaching Time~~ Estimates of the Effort Devoted to the Teaching of Each Principle At Each Grade Level.	VI-13
VI-2	Statistically Significant Post-Test Differences Between Students Who Were Taught Logic and Students Who Were Not Taught Logic, Using Analysis of Covariance with Pre-Test and I.Q. as Covariates.	VI-20
VI-3	Conditional Reasoning Pre- and Post Differences for 11th Graders to Whom Logic Was Deliberately Taught	VI-22
VI-4	Conditional Reasoning Pre-and Post- Test Differences for Principle #1, the Affirmation-of-the-Antecedent Principle:	VI-23
VI-5	Multiple Correlations, Cross Validation Estimates, and Standard Errors for Prediction Equations.	VI-33
VII-1	A Tentative Time Table for Future Phases of the Critical Thinking Readiness Study.	VII-7
A-1	Individual Item and Item-Group Mean Difficulty Indices by Grades on the Class Reasoning Test for All Subjects on the Pre-Test and also for LDT Pre- and Post-Tests.	Appendix
A-2	Individual Item and Item-Group Mean Difficulty Indices by Grades on the Conditional Reasoning Test For All Subjects on the Pre- Test and also for LDT Pre- and Post-Tests.	"

[redacted]		Page
A-3	Individual Item and Item-Group Mean Discrimination Indices by Grades on The Conditional Reasoning Test For all Subjects on the Pre-Test.	Appendix
A-4	Individual Item and Item-Group Mean Discrimination Indices by Grades on the Class Reasoning Test for All Subjects on the Pre-Test.	"
A-5	Conditional Reasoning Adjusted-Mean Post-Test Comparisons of Students to Whom Logic Was Deliberately Taught and Students to Whom Logic Was Not Deliberately Taught.	"
A-6	Class Reasoning Adjusted-Mean Post-Test Comparisons of Students to Whom Logic Was Deliberately Taught and Students to Whom Logic Was Not Deliberately Taught.	"
A-7	Pre- and Post-Test Mean Difficulty Indices for Students to Whom Conditional Reasoning Was Deliberately Taught, by Item Groups and Grades.	"
A-8	Pre- and Post-Test Percentages of LDT's Who Met the Sufficient Condition for Mastery, and Percentages Who Failed to Meet the Necessary Condition for Mastery of Principles of Conditional Reasoning, By Item Group and Grade.	"
A-9	Pre- and Post-Test Mean Difficult Indices for Students to Whom Class Reasoning Was Deliberately Taught, by Item Groups and Grades.	"
A-10	Pre- and Post-Test Percentages of Students Who Were Deliberately Taught Class Reasoning Who Met the Sufficient Condition For Mastery and Percentages Who Failed to Meet the Necessary Condition, by Item Group and Grade.	"
A-11	Conditional Reasoning Weights and Constants for Multiple Regression Equations Making Use of Pre-Tests.	"
A-12	Conditional Reasoning Weights and Constants for Multiple Regression Equations Not Using Pre-Test Scores.	"
A-13	Class Reasoning Weights and Constants for Multiple Regression Equations Making Use of Pre-Test.	"
A-14	Class Reasoning Weights and Constants for Multiple Regression Equations Not Using Pre-Test Scores.	"

LIST OF GRAPHS

		Page
V-1.	Conditional and Class Reasoning Mean Total Score vs. Mean Chronological Age.	V-21
V-2.	Conditional and Class Reasoning Mean Total Score vs. Mean Mental Age.	V-23
VI-1	Empirically Derived Relationships Between Mean Difficulty Indices and the Percentages Meeting the Sufficient Condition and Failing to Meet the Necessary Condition.	VI-35
A-1	Plotted Points: Conditional Reasoning Mean Difficulty Index vs. Percentage Meeting the Sufficient Condition	Appendix
A-2	Plotted Points: Conditional Reasoning Mean Difficulty Index vs. Percentage Failing to Meet the Necessary Condition.	"
A-3	Plotted Points: Class Reasoning Mean Difficulty Index vs. Percentage Meeting the Sufficient Condition.	
A-4	Plotted Points: Class Reasoning Mean Difficulty Index vs. Percentage Failing to Meet the Necessary Condition.	"

Chapter I - Introduction

Many curriculum reforms have in the history of our schools been accompanied by confusion about just what they were supposed to accomplish and by ignorance about the feasibility of trying to accomplish whatever it was they were doing at the level at which they were trying to do it.

In instituting the Critical Thinking Readiness Project it was our hope to do what we could to help avoid these two dangers in the attempts to introduce instruction in critical thinking. Our primary goals in this project are to contribute to knowledge about what critical thinking is and to knowledge about when it can be taught. This report, covering the first phase in our attempts to achieve these goals, deals with deductive logic - and adolescents (age 10-18).

A more specific list of the purposes of this phase follows. Some of these purposes are subsidiary and some are complementary to the above-mentioned pair of goals, as limited for this first phase.

1. To become more clear about the nature of deductive logic, as used in ordinary reasoning and to compare this analysis with Piaget's to see whether we were talking about the same thing.
2. To find out when students are ready to master principles of deductive logic.
3. To be clear about the concept, readiness to master a principle.
4. To build logic tests suitable for use in this study.
5. To provide an operational interpretation of 'mastery of a principle'.
6. To investigate the degree of mastery of principles of logic currently found among students of ages 10-18.

In this report there are no definitive answers to the questions implied by those purposes, but two general principles have clearly emerged as a result of our efforts:

1. In this area of study there are many conceptual pitfalls for the unwary.

2. Answers to the questions implied by the above list of purposes are of the 'that depends' form. For example, the answer to the question about when students are ready to learn deductive logic depends not only upon the type of logic, but also upon the principle of logic within that type. In his developmental studies of knowledge of logic, Piaget distinguished only among types of logic, making rather broad statements about these types.

This report of our efforts to achieve these purposes is divided into eight chapters. Each chapter is fairly self-contained, but is best understood in conjunction with the others. Probably the best order of reading is to start with this, the introductory chapter; then to read the last, the summary chapter; and then to read the other chapters in order (Chapters II to VII). The middle chapters, which make up the body of the report, are in a plausible sequential order. Here is a list of the questions dealt with in Chapters II through VII, and a list of the corresponding chapter headings:

	Questions	Chapter Headings
I.	What is to be found in the report?	INTRODUCTION
II.	What is the nature of the logic content we were trying to convey and how does this compare with Piaget's notion of logic?	THE SUBJECT MATTER: LOGIC
III.	Who were our subjects and how did we secure information about them?	BASIC DATA ON SUBJECTS
IV.	What is the nature of the tests used to measure knowledge of logic, and how did we operationally interpret 'mastery of a principle of logic'?	THE CORNELL DEDUCTION TESTS
V.	What sort of development of knowledge of logic occurs without deliberate instruction in school, that is, as a result of what we have called "natural-cultural influences"? How does this development vary for different principles of logic, and how does it vary with the content of the propositions used in reasoning?	THE NATURAL-CULTURAL DEVELOPMENT OF KNOWLEDGE OF LOGIC

	Questions	Chapter Headings
VI.	What principles of logic were our subjects ready to master, given 15 daily periods of instruction in logic? And what sorts of content were they ready to reason with? What are other students ready to master, given comparable instruction? What is the meaning of 'readiness for mastery of a principle'?	THE DEVELOPMENT OF READINESS TO MASTER LOGIC
VII.	What happened in prior years that led up to the work directly reported on here. What is next?	THE PAST AND FUTURE PLANS OF THE PROJECT
VIII.	Briefly what appears in this report?	SUMMARY

In organizing this report we considered having a chapter on the statement of the problem, one on related research, one on procedures, one on analysis of data, etc., but decided against this approach because it would fragment the focuses of concern. Instead each chapter contains, if appropriate, a section clarifying the problem, a section on related literature, a section analyzing key concepts, etc. This we believe gives the report greater unity than it otherwise would have and makes for less page turning on the part of the careful reader.

In much of the work reported on here, we were exploring virgin territory, and found that we were spending a good deal of time on questions prior to the actual standard experimental activities of doing and observing. These prior questions involved clarification of questions and concepts and their operational interpretation (including test construction). Although our empirical findings are important, perhaps the greater contribution of this study lies in the suggestion of ways of approaching the readiness question, and in the clarification of concepts basic to the empirical work. Actually this study only opens the door to a vast array of investigations.

Chapter II. The Subject Matter: Logic

In this chapter we shall first examine the nature of our general concern, critical thinking, indicating the role played by deductive logic; then we shall examine deductive logic itself, discussing the nature and importance of various types, and comparing the logic we taught with that which Piaget investigated. Throughout we shall attempt to avoid decisions that commit us to a stand on some of the contemporary issues in the field of logic. Although we do have opinions about many of these issues, the actual criteria that we use (especially our test items) should be acceptable to the major contending positions in contemporary logical theory.

A. AN ANALYSIS OF CRITICAL THINKING

Our analysis of critical thinking springs from the basic notion put forward by B. Othanel Smith: "Now if we set about to find out what...[a] statement means and to determine whether to accept or reject it, we would be engaged in thinking which, for lack of a better term, we shall call critical thinking." (1953, p. 130). Note that this notion of Smith's implies a distinction between critical and creative thinking. In critical thinking, the item to be evaluated has already been produced.

This distinction between critical and creative thinking enables one to divide in two parts the problem of teaching people to be good thinkers. Each part is of course quite important and in practice the parts are quite interdependent. But for purposes of careful investigation, it is helpful to separate out this part (critical thinking), for the judgment of which there are already fairly well established criteria.

A minor difference between our notion of critical thinking and that which Smith presents can be found in the fact that we have included the idea of proficiency in ours, whereas Smith does not require that the determination about whether to accept or reject be done properly or well in order that critical thinking take place. We believe that our notion is more in accord with everyday usage, but feel that so long as one is clear about which notion he is using and makes sure that the rest of the theory fits the notion selected, either notion is workable.

Accepting this basic notion as revised, the principal investigator has been developing and refining a list of abilities which characterize the critical thinker. The following list represents the current state of this investigation. Although it is fairly self-explanatory, one might want to seek clarification in the selection, "A Definition of Critical Thinking" (Ennis, 1964a), or in the more theoretical analysis, "A Concept of Critical Thinking" (Ennis, 1962).

A critical thinker is characterized by proficiency in judging whether:

1. A statement follows from the premises.
2. Something is an assumption.
3. An observation statement is reliable.
4. A simple generalization is warranted.
5. A hypothesis is warranted.
6. A theory is warranted.
7. An argument depends on an ambiguity.
8. A statement is overvague or overspecific.
9. An alleged authority is reliable.

Although the basic notion, as revised, calls for the inclusion of proficiency in judging value statements, for the time being that proficiency is excluded from our list because value statements constitute one area where fairly well established and agreed upon criteria do not exist.

Deductive logic, the subject of the current study, is a central part of critical thinking, as analyzed above. First of all, since it deals with the question of whether a statement follows necessarily from another statement or statements, it is at least central to the first of the nine aspects listed above.

However, though opinions may differ on this matter, deductive logic does not seem to us to exhaust this aspect of critical thinking. Although logic is a guide, the validity of most real arguments appears to us to depend on other considerations. The connection between the premises and conclusion does not seem as strict as that in the model provided by deductive logic. The correctness of this claim about the relation between premises and conclusions and the nature of the relationship that does exist are important topics for philosophical research. But regardless of the outcome of this research it must still be granted that deductive logic plays at least a major role in the first aspect of critical thinking.

Secondly deductive logic plays an important role in the other aspects as well. Among other things, it is a constituent of the application of the criteria and/or principles of the other eight aspects.

Primarily because of its centrality in critical thinking, but also because its criteria seemed to be in more suitable shape than the criteria for other aspects of critical thinking, we picked deductive logic as the first thing to be investigated. It is our hope to perform and/or to stimulate parallel and complementary investigations of other aspects of critical thinking.

B. TYPES OF DEDUCTIVE LOGIC

For purposes of this study the following explanation of the meaning of 'deductive logic' will suffice: Deductive logic is concerned with whether a statement follows necessarily from one or more other statements. A statement follows necessarily, if, and only if, its denial contradicts the assertion of the other statement(s).

Three recognized types of deductive logic are sentence logic, class logic, and ordinal logic. These types are often called by other names, the name depending sometimes on one's philosophy of logic, but these names will serve to identify three types of logic which are often so grouped.

1. Sentence Logic.

Sentence logic is concerned with arguments in which the basic units are sentences. That is, distinct sentences, often connected or modified by such logical connectives as 'if', 'only if', 'then', 'and', 'or', 'not', and 'both', appear essentially unchanged throughout the course of the argument. Here is an example taken from "The Cornell Conditional Reasoning Test":

Suppose you know that

> Tom may use paints only if he has cleaned up his clay work.
> [and]
> Tom may use paints.

Then would this be true?

> Tom has cleaned up his clay work.

In that series the two sentences, "Tom may use paints" and "Tom (or he) has cleaned up his clay work", each appear twice. In the first of the two given statements they are joined by the logical connective, "only if", but these two basic sentences appear essentially unchanged throughout the course of the argument. Hence this is a case of concern for sentence logic.

Sometimes sentence logic itself is broken up into parts, depending on the logical connective which is used. When the connective is 'if', 'only if', or 'if, and only if', or any synonyms of these, we have what is sometimes called, and what we shall call, a 'conditional statement'. Arguments which contain only conditional statements and simple sentences or negations thereof shall be called conditional arguments. Reasoning associated with such arguments shall be called conditional reasoning.

Other types of sentence logic are called by different systems of names. One consistent set labels as 'alternation statements' those which make use of the connective, 'or'; 'disjunctive statements' those which use the connective 'not...both';

and 'conjunctive statements' those which contain the conjunction 'and' (Cohen and Nagel, 1934). A more common set of labels at the present time applies the word 'disjunction' to statements containing 'or' instead of those containing 'not... both', and considers the latter simply denials of conjunctions. Although we did not have to make a choice for the present research project, a choice will have to be made in the continuation into these areas. The choice is not simply an arbitrary one, since it depends to some extent on one's philosophy of logic. Our current inclination is to recommend the former system to people investigating these areas in the future, because the connection implicit in a 'not...both' statement appears to be more fully recognized with the separate name 'disjunction', and because the names of the former system seem more natural.

2. Class Logic.

The familiar traditional syllogisms are arguments in class logic form, but they do not exhaust class logic arguments, so a more general description is necessary.

The basic units in class logic are parts of sentences, subjects and predicates. The sentences do not reappear essentially unchanged; instead the subjects and predicates are separated from each other and rearranged. Here is an example to which the criteria of class logic are to be applied. It is from "The Cornell Class-Reasoning Test":

Suppose you know that

All the people who live on Main Street were born in Milltown.
None of the students in Room 352 live on Main Street.

Then would this be true?

None of the students in Room 352 were born in Milltown.

For purposes of simplification and ease of teaching, the subject and predicate are revised in order to form classes. Following this procedure the classes involved in the above example are 1) the people who live on Main Street, 2) the people who were born in Milltown, and 3) the students in Room 352. The two given statements present relationships between the first and second classes and the third and second classes respectively. The statement about which one must decide suggests a relationship between the third and first classes. Thus the subjects and predicates as represented by the classes are the basic units in this kind of reasoning.

Admittedly the representation of subjects and predicates by classes is a simplification which can result in philosophical problems. However, as long as one is cautious enough to avoid thinking that the class relationships somehow more suitably capture the meaning than the original statements, the transformations can be convenient in explanation and teaching.

3. Ordinal Logic.

A third type of logic deals with size relationships, such as, greater than, equal to, less than, not greater than, etc. Here is an example taken from Burt's "Graded Reasoning Tests" (1919):

> Tom runs faster than Jim; Jack runs slower than Jim. Who is the slowest -- Jim, Jack, or Tom?

4. Other Types of Logic.

No successful comprehensive classification system of types of logic has even been prepared, so the best we can do here is to list some other types with an explicit disavowal of comprehensiveness.

Mathematical reasoning is such that the conclusion is supposed to follow necessarily from the premises and would therefore be classified as logic. Since we have no desire to enter the jurisdictional dispute (about which is a branch of which) between some logicians and some mathematicians, we simply point out the consequences of our definition, being ready to accept an amendment, if desired. Our definition, however, should at least serve to alert us to the similarities between the fields of mathematics and logic.

Several other branches of logic with which at least some work has been done by logicians are, using the terminology suggested by G. H. von Wright

(1957, p. 58), alethic logic (traditionally called 'modal logic'), epistemic logic, and deontic logic. Alethic logic is concerned with statements of possibility and necessity; epistemic logic with knowledge statements; and deontic logic with statements of obligation. Each of these branches of logic is important in critical thinking, but no one of them has yet been sufficiently worked out for us to do research on them of the sort which is being described in this report. Hopefully the current trend among philosophers toward increased interest in these fields will continue, so that some satisfactory criteria will be available for use in research like ours.

Presumably other types of logic could yet be identified, because no one has successfully presented a rationale for the exhaustiveness of some list, and it has been our experience that numerous examples of inferences that seem in a way necessary do not clearly and uncontroversially fit any of the types mentioned. Applications of broad value statements and inferences to and from statements containing such words as 'probably' and 'because' are the sorts of thing we have in mind.

There is thus still much philosophical spadework to be done, both in identifying types of logic and in determining valid patterns of inference for the relatively unexplored types. Until this philosophical spadework is done, the investigation of patterns of development, learning, and learning capacity in the types of logic must remain unfinished.

C. AN ELABORATION OF THE TYPES OF LOGIC INVESTIGATED IN THIS STUDY

Though an investigation of all types of logic is not yet possible, we were able to work with two very common and significant types of logic, class logic and conditional logic. We picked these for several reasons:

1. The criteria for judging arguments in these forms of logic are fairly well developed and agreed upon. Where controversy does exist,* we essentially avoided it.

---

*See P. F. Strawson's Introduction to Logical Theory (1952) for an indication of areas of controversy that do exist.

2. These two types of logic, together with the other kinds of sentence logic, are the things most commonly taught under the heading, "deductive logic", are thus well associated with the term, 'deductive logic'. We felt that we were forced to select from among the branches of sentence logic because of time pressures, and picked conditional logic because the if-then relationship is fundamental in all logic and because an understanding of conditionals together with conjunction and negation enables one to do other types of sentence reasoning. This selection of conditional logic from sentence logic should be kept in mind as you read the next two reasons for our selection of certain types of logic, because they argue for the inclusion of sentence logic instead of only conditional logic.

3. An informal investigation that we performed of reasoning in newspaper editorials, U. S. Supreme Court opinions, and an auto mechanics handbook showed that class logic and sentence logic included most of the deductive reasoning that occurred there. The only notable exception was deontic logic, the logic of obligation.

4. In the literature on people's reasoning there is some precedent for the selection of class logic and sentence logic. Most of the studies that we have located deal with one or both of these types.

5. Although ordinal logic is well worked out, is already being taught in schools, and is considered in the literature, we omitted it. We did include it in our pilot study, but felt that in order to do a more satisfactory job in the year 1963-64, we had to reduce the scope of our project from three types of logic to two. Partly because ordinal logic is inevitably receiving some readiness attention now; partly because our pilot study showed that major advances in capacity for mastery of the principles of ordinal logic occurred in the primary grades (ages 6-12), especially early primary (ages 6-9) for most of the students we studied; and lastly because work of the type we were doing requires more personnel for younger children, we decided to omit ordinal logic in the study

being reported on here.*

Given class and conditional logic as our concerns, a fuller account of each is in order. In what follows we shall set forth and discuss what we consider to be the basic principles of each. These lists of principles are not in the most elegant form (for the more elegant they are, the less easy they are for most people to understand), nor are they in the most easily understandable form, but are in a form that is something of a compromise between the two.

In our teaching of logic we did not use the language of these statements of principles, because we felt much of it to be too formidable for teaching purposes. This is so especially for class logic, in the teaching of which we made use of a system of circles which provided a model for class relationships.

With but one exception the principles are tested for in our tests, which will be described later. This exception, Conditional #12, was omitted for reasons given in Chapter IV, which describes the tests. Some combinations of these principles are also tested for in our tests. These combinations are not listed here. They will be indicated in Chapter IV.

In the hope that they will be self-explanatory, these lists are arranged as follows: On the left the principle is stated in English. In the center and on the right arguments appear. In each case the principle justifies a judgment of valid (conclusion follows necessarily) or invalid (conclusion does not follow necessarily) with respect to the argument. The argument in the center is in symbolic form. The argument on the right is an example taken from one of our tests and modified for the purposes of this method of presentation.

The conditional logic principles are ordered in our recommended order of classroom presentation. This ordering is based upon our experience teaching logic, but has not been subjected to experimental test.

---

* Professor Lucille Ringel of Fenn College, Cleveland, is preparing a report of the work that we did do with ordinal reasoning.

TABLE II-1. Illustrated Basic Principles of Conditional Logic

Principle	Symbolized Argument	Concrete Argument
1. Given an if-then sentence, the affirmation of the if-part implies the affirmation of the then-part.	If p, then q. p. Therefore q. Valid.*	If the hat on the table is blue, then it belongs to Joan. The hat on the table is blue. Therefore the hat on the table belongs to Joan.
2. Given an if-then sentence, the denial of the if-part does not by itself (as a result of its being an if-part) imply the denial of the then-part.	If p, then q. Not p. Therefore not q. Invalid.	If Tom lives in the white house, then his last name is Smith. Tom does not live in the white house. Therefore Tom's last name is not Smith.
3. Given an if-then sentence, the affirmation of the then-part does not by itself (as a result of its being a then-part) imply the affirmation of the if-part.	If p, then q. q. Therefore p. Invalid.	If Mary lives in the white house, then her last name is Brown. Mary's last name is Brown. Therefore Mary lives in the white house.
4. Given an if-then sentence, the denial of the then-part implies the denial of the if-part.	If p, then q. Not q. Therefore not p. Valid.	If the car in the parking lot is Mr. Smith's, then it is blue. The car in the parking lot is not blue. Therefore the car in the parking lot is not Mr. Smith's.
5. The if-then relationship is transitive.	If p, then q. If q, then r. Therefore, if p, then r. Valid.	If Sam misses the bus, he will walk to school. If Sam walks to school he will cross the bridge. Therefore, if Sam misses the bus, he will cross the bridge.

---

* The validity status of the arguments will only be specified for the symbolized ones. It is the same in each case for both the symbolized and concrete arguments.

TABLE II-1 (cont.)

6. An if-then sentence implies its contrapositive.	If p, then q. Therefore, if not q, then not p. Valid.	If Mrs. Smith entered the flower show, then she entered her roses. Therefore, if Mrs. Smith didn't enter her roses, then she didn't enter the flower show.
7. The if-then relation is non-symmetric.	If p, then q. Therefore, if q, then p. Invalid.	If the chair is green, then the table is black. Therefore, if the table is black, then the chair is green.
8. Given an only-if sentence, the denial of the only-if part implies the denial of the major part.	p only if q. Not q. Therefore not p. Valid.	John is in the kitchen only if there is food in the kitchen. There is no food in the kitchen. Therefore John is not in the kitchen.
9. Given an only-if sentence, the affirmation of the major part implies the affirmation of the only-if part.	p only if q. p. Therefore q. Valid.	Harry is on the football team only if he has his mother's permission. Harry is on the football team. Therefore Harry has his mother's permission.
10. The denial or affirmation of one part of an if-and-only-if statement implies respectively the denial or affirmation of the other part.	p, if, and only if, q. Not p. Therefore not q. Valid.	Bill will see Audrey this year, if, and only if, he goes to Montreal this year. Bill will not see Audrey this year. Therefore, Bill is not going to Montreal this year.
11. Given an only-if sentence, the affirmation of the only-if part does not by itself (as a result of its being an only-if part) imply the affirmation of the major part.	p only if q. q. Therefore p. Invalid.	Dick is using the classroom dictionary only if the library is closed. The library is closed. Therefore Dick is using the classroom dictionary.
12. Given an only-if sentence, the denial of the major part does not by itself (as a result of its being the major part) imply the denial of the only-if part.	p only if q. Not p. Therefore not q. Invalid.	Jane went to the park yesterday only if she saw her friend Pat yesterday. Jane did not go to the park yesterday. Therefore Jane did not see her friend Pat yesterday.

TABLE II-2. Illustrated Basic Principles of Class Logic

Principles	Symbolized Arguments	Concrete Arguments
1. Whatever is a member of a class is not a non-member of that class and vice versa.	All A's are B's. Therefore it is false that at least some A's are not B's. Valid.	All of John's pencils are blue. Therefore it is false that at least some of John's pencils are not blue.
2. Whatever is a member of a class is also a member of a class in which the first is included. (This implies that class inclusion is transitive.)	All A's are B's. All B's are C's. Therefore all A's are C's. Valid.	All the cars in the garage are Mr. Smith's. All Mr. Smith's cars are Fords. Therefore all the cars in the garage are Fords.
3. Whatever is a member of a class is not (as a result of that relationship) necessarily a member of a class included in that class.	All A's are B's. Therefore all B's are A's. Invalid.	All the red books are John's. Therefore all John's books are red.
4. Class exclusion is symmetric.	No A's are B's (and there are B's). Therefore no B's are A's. Valid.	None of Jane's dolls have hats. Therefore none of the dolls that have hats are Jane's.
5. Whatever is a member of a class is not a member of a class excluded from the first.	All A's are B's. No B's are C's. Therefore no A's are C's. Valid.	All Frank's homework is due today. None of the homework due today is in history. Therefore, none of Frank's homework is in history.
6. Whatever is not a member of a class is not (as a result of that relationship) necessarily also not a member of a class in which the first is included.	All A's are B's. No C's are A's. Therefore no C's are B's. Invalid.	All the papers in the box are torn. None of John's papers are in the box. Therefore none of John's papers are torn.

TABLE II-2 (cont.)

7. Whatever is not a member of a class is not (as a result of that relationship) necessarily a member of (nor a non-member of) another class which is excluded from the first.	No A's are B's. No C's are B's. Therefore at least some A's are C's. Invalid.	None of my shirts are wool. None of the shirts hanging up in the closet are wool. Therefore, at least some of my shirts are hanging up in the closet.
8. Whatever is not a member of a class is also not a member of any class included in the first.	All A's are B's. No C's are B's. Therefore no C's are A's. Valid.	All of Joan's homework is due today. None of the math homework is due today. Therefore none of the math homework is for Joan.

On the other hand the principles of class logic are not strictly in a recommended teaching order, since our teaching method for class logic makes use of the circle system mentioned earlier. But we think that the first four of these principles are grouped roughly in teaching order anyway.

A few words of summary of these principles are in order. The conditional principles cover affirming and denying the antecedent ("if-part" and "major part") and consequent ("then-part" and "only-if-part") in both if-then form (1-4) and only-if form (8,9,11,12). Transitivity (5), contraposition (6), non-symmetry (7), and the combination of 'if-then' and 'only if' (10) are the other things covered by the conditional logic principles. A person familiar with conditional logic will recognize these as the basic elements of conditional logic, though he might of course feel some redundance if he seeks logical elegance. It is our view that in spite of this sort of redundance, the principles must at least for teaching purposes be spelled out to the extent that we have done so. And then there is the very difficult question of whether they really are redundant anyway, because the different forms (as we have specified them) are used under different circumstances. Fortunately we do not have to settle the question because teaching requirements force the large number of principles on us.

The class principles cover first of all the basic meaning of being and not being a member of a class (1), and secondly the basic notion of two classes' being excluded from one another (4). Then the main body of principles falls into two groups: those starting with the assumption that something is in* a class (or part of a class), (2,3,8) and those starting with the assumption that something is not in a class (or part of a class), (5,6,7). Each group is then subdivided into three parts according to whether that class (or part of a class) is a) included in a third (2,6), b) includes a third (3,5) or c) is excluded from a third (8,7). Thus the possibilities are all covered.

---

* Deliberately vague to cover both membership and inclusion.

We regard class principles numbered 2 and 3 as the two most important to teach to someone if he does not know them. Number 2 catches the transitivity of inclusion while number 3 catches its non-symmetry.

A good way to grasp the impact of the class principles, if the arguments do not suffice as explanation, is to use circles inside one another or separate from one another to represent the inclusion or exclusion relationships.* A conclusion inescapably diagrammed by the diagramming of the premises follows necessarily. If not inescapably diagrammed, then it does not follow necessarily.

That completes our direct characterization of the logic that we taught. The next section, which compares this logic with Piaget's, indirectly provides some further characterization.

D. A COMPARISON WITH PIAGET'S LOGIC

Jean Piaget is the leading figure in the study of the development of children's knowledge of logic, though not their capacity to learn logic, which was in addition one of our major concerns. Since a comparison with his conclusions is inevitably called for, it is important to try to be clear about whether what Piaget calls 'class logic' and 'propositional logic' correspond to what we have called 'class logic' and 'sentence logic', because only to the extent that they correspond does the question of agreement or disagreement about the facts of development arise. In some ways the logics seem to correspond and in some ways they do not seem to do so. In fact it is difficult to be sure of the extent of correspondence. This might be explainable by Piaget's difficult style; and it might be explainable by changes over time in his concepts to adjust to the facts or his own developing interests. Regardless of the explanation, the points of correspondence and lack of correspondence, as we see them, follow. We must confess that we have not been able to construct with confidence what we consider to be a coherent consistent account of Piaget's logic, but we shall do the best we can. Discussions of

---

*This is a simplified account, as you might expect.

Piaget's logic have also been presented by Parsons (1960) and Flavell (1963).

Points of Correspondence.

First of all Piaget did talk about logic. Secondly he does distinguish between two types of logic, and thirdly this distinction basically seems to correspond to the distinction between class logic and sentence logic that we drew earlier in the chapter. We shall expand on each of these points.

1. Piaget seems to have adopted some of the basic moves and definitions in contemporary propositional logic (1958*, pp. 293-303). His logical operators, 'v', '⊃', '.', and '-', correspond to the operators of traditional propositional logic, and stand respectively for 'or', 'if..., then', 'and', and 'not'. He makes use of the same letters as are traditionally used to represent propositions, 'p', 'q', and 'r'. And he generally relates all these in a way that essentially conforms to that of any elementary text in symbolic logic.

Furthermore his class logic symbolism, definitions, and rules are in conformity with at least much of what is traditionally classified as class logic. Among other things, he uses capital letters to represent classes and uses a single quote after a letter which represents a class to represent the complement of that class (1958, pp. 274-77).

Hence it is quite clear that Piaget was talking about logic, establishing at least some correspondence between his interests and ours.

2. Along with us, Piaget resists the contemporary trend toward the merging of propositional and class logic. This merging is accomplished by treating class inclusion statements as modified conditionals. For example, the statement, 'All floating bodies are light', is transformed into the statement, 'For every x, if x is a floating body, then x is light'.

---

* Henceforth in this section on Piaget, all references made by the use of '1958' shall be to Inhelder and Piaget's *The Growth of Logical Thinking From Childhood to Adolescence*. We believe that the parts to which we refer were written by Piaget and not Inhelder.

Sometimes the transformation goes further, putting the statement in terms of existence, conjunction, and negation, making the above statement look like this: 'It is not the case that there exists an x such that x is a floating body and x is not light.'

One of Piaget's reasons, with which we are in sympathy, is that to so interpret class inclusion statements would be to use "a complex language for describing phenomena which do not go beyond much simpler structures in the subject's mind" (1958, p. 280).

3. A third point of correspondence lies in the similarity of his basis for the distinction between propositional and class logic and our basis for the distinction between sentence and class logic. Piaget says that although class logic does deal with propositions, "decomposing and recomposing the content of propositions" (1958, p. 292), it does not deal with the combination of these propositions as independent units. Propositional logic, on the other hand, does this.

In another place Piaget in characterizing the formal operational period by a person's ability to do propositional logic, says, "Formal operations, therefore, consist essentially of 'implications' ... and 'contradictions' established between propositions which themselves express classifications, seriations, etc." (1950, p. 149).

Thus for Piaget, as for us, class logic is concerned with the internal features of propositions which are not themselves composed of other propositions, whereas propositional (and sentence) logic is concerned with the relations between propositions (or sentences) which themselves remain essentially unchanged throughout an argument.

We do not feel that the difference in names ('propositional' vs. 'sentence') constitutes a significant difference between Piaget and us. His term, 'propositional' is in more common use among logicians; for that reason we would have chosen it if we had not felt that communication with teachers and students will be facilitated by the use of the term 'sentence', which already has meaning for them. The term 'proposition' in the sense in which it is used by logicians is quite unfamiliar to most people.

Now it is the case that the two terms can be used to mark different positions in philosophy of logic, but the use of one term or the other does not necessarily commit one to one of the positions, since the choice of one term or the other is often simply a matter of convention or convenience. The positions that can be marked by these terms are positions about the nature of the basic units that are connected and modified by the logical connectives, 'or', 'if..., then', 'and', 'not', etc. Are the basic units merely strings of words concocted by human beings, or are they possessed of some sort of independent existence, regardless of whether they are formulated by human beings?

We do not want to take a stand on this issue; we do not even want to take a stand on whether or not it is a genuine issue. But we do want to use the term 'sentence' because of its established usage. We do not know whether Piaget takes a stand on the issue, but feel that as far as the teaching of the rules of logic goes, it does not matter. His difference in usage does not represent a substantive difference.

Points of Noncorrespondence.

There are two points of noncorrespondence which we would like to suggest: First the judgment about whether propositional or class logic is in use seems at times to depend for him upon whether there is a consideration of all the possibilities inherent in the situation. Secondly Piaget's system is more simple than the one we worked with. We are less sure about the first difference than the second.

1. According to Piaget (1958, pp. 272-333) an important feature of a person's reasoning is the extent to which he works within a system of all the possible combinations of the variables and views what he sees as one of these

possible combinations. Associated with this, according to Piaget, is the subject's ability to separate out the variables. Presumably if a person separates out variables and holds certain ones constant, then he is eliminating certain of the possible combinations of variables as possible causes of the phenomena being investigated.

There is a strong suggestion that a defining characteristic of ability at propositional logic is ability to work within a system of all possible combinations. Furthermore a defining characteristic of the use of class logic is the failure to work within a system of all possible combinations. These are not defining characteristics of our sentence and class logics.

He says, in searching for a way of telling "in which cases the subjects reasoned through arrangements of classes and relations and in which cases they used propositional operations" (1958, p. 279), "It is fruitless to look for an exclusively verbal or linguistic criterion -- e.g., considering all statements containing the words 'if...then' as implications while regarding the statements which do not contain them as inclusions or correspondences, etc." (1958, p. 279). He then says that a better method of making the judgment about whether class or propositional logic is in use is look at all the actions of a subject and see:

> whether he tries to separate out the variables, [which] implies both hypothetico-deductive reasoning and a combinatorial system; when they appear, we have to interpret the stated judgments as propositional expressions ... (1958, p. 279).

Thus separating out the variables seems to be a criterion of the use of propositional logic.

But the "surest method of differentiation", according to Piaget is to see if the subject interprets a given correspondence as the result of any one of several possible combinations:

> If...the subject interprets a given correspondence as the result of any one of several possible combinations, and this leads him to verify his hypotheses by observing their consequences, we know that propositional operations are involved (1958, p. 279).

It is because of these statements and statements like them that we suspect that Piaget holds that a definitionally necessary condition for the use of propositional logic is that of working within a system which contains all the possible combinations of the variables, and that a definitionally necessary condition for the use of class logic is the lack of working within such a system.

What it is to work within such a system still remains to be clarified and unfortunately Piaget does not make himself fully clear. Here is the best account that we can work out:

According to Piaget, given two propositions (or classes), there are sixteen possible ways in which they can be grouped. For propositions he calls these ways the "sixteen binary operations" (1958, p. 293) and for classes he simply lists them as "sixteen possible combinations" (1958, p. 277). The two lists correspond, as he indicates in a series of footnotes in his discussion of the sixteen binary operations (1958, pp. 293-303), so we will present only the list of the sixteen binary operations.

In this list 'p' and 'q' stand for propositions. A denial of the proposition is represented by the symbol with a line over it, e.g., '$\bar{p}$'. A conjunction of two propositions is shown by putting a dot between them e.g., 'p.q' which means 'p and q'. 'Either p or q' is represented by 'p v q', and 'If p, then q' is represented by '$p \supset q$'.

The sixteen binary operations:

1. $p.q \vee p.\bar{q} \vee \bar{p}.q \vee \bar{p}.\bar{q}$
2. The negation of #1.
3. $p.q$
4. The negation of #3: $\overline{p.q}$
5. $p \vee q$
6. The negation of #5: $\bar{p}.\bar{q}$
7. $p \supset q$
8. The negation of #7: $p.\bar{q}$
9. $q \supset p$
10. The negation of #9: $\bar{p}.q$
11. $(p \supset q).(q \supset p)$
12. The negation of #11: $p.\bar{q} \vee \bar{p}.q$
13. $p.(q \vee \bar{q})$
14. The negation of #13: $\bar{p}.(q \vee \bar{q})$
15. $q.(p \vee \bar{p})$
16. The negation of #15: $\bar{q}.(p \vee \bar{p})$

A person acquainted with symbolic logic can see that (given the symbolic logic interpretation of these symbols) this list contains the affirmation and denial of each of the possible groupings of p and q and their denials. For him who is not well acquainted with symbolic logic the exhaustiveness of the combinations that Piaget sees perhaps can be shown thusly:

Let 'a' stand for '$p.q$'
Let 'b' stand for '$p.\bar{q}$'
Let 'c' stand for '$\bar{p}.q$'
Let 'd' stand for '$\bar{p}.\bar{q}$'

These are the four possible conjunctions of the assertion and denial of 'p' and 'q'. Then there are sixteen possible ways that 'a', 'b', 'c', and 'd' can be grouped:

1. a
2. b
3. c
4. d
5. ab
6. ac
7. ad
8. bc
9. bd
10. cd
11. abc
12. abd
13. acd
14. bcd
15. abcd
16. 0 (that is, none of them)*

If one accepts the assumptions, one can see the exhaustiveness of the system of combinations of the assertion and denial of the two propositions.

Now we suspect that for Piaget a person is not doing propositional logic unless he is operating within a framework that will take account of all of these possibilities. If this suspicion is correct, then Piaget's logic does not correspond to the logic that we taught, nor does it correspond to the standard contemporary interpretation. The latter might say that a person is not fully proficient if he does not operate within such a framework, but it would not deny the application of the term, 'propositional logic', to the following argument, if advanced by a person who was not aware of nor working within a complete combinatorial framework:

---

*This system of explanation is suggested by Flavell (1963, pp. 213-14).

If p, then q. But not q. Therefore not p.

We cannot be sure whether this interpretation of Piaget is correct, because it does not seem compatible with the distinction between propositional and class logic which we presented under "Points of Correspondence". If the interpretation is correct, then there is a vast difference between the two logics.

2. A second point of noncorrespondence, one of which we are confident, is the greater simplicity of the system that he advocates. His system is more simple in several different ways:

a. The list of sixteen binary operations together with the interpretation given to the symbols shows that Piaget is accepting the reduction of the various logical operators to two: conjunction and negation.* For example, for him, 'If p, then q' means the same as 'It is not the case that both p and not q'. Symbolically, '$p \supset q$' = '$\overline{p \cdot \bar{q}}$'.

The acceptability of this reduction is a contemporary issue in logical theory. Common practice is to accept the above symbolic equivalence, but to deny that '$p \supset q$' captures the meaning of 'If p, then q'. Piaget makes no such reservations, so far as we can tell. In case any of our readers would like to pursue this issue we mention the fact that prominent contenders are P. F. Strawson and P. H. Grice, both of Oxford University. Strawson opposes the reduction and Grice tends toward supporting it.**

---

* Another way of looking at it is to say that the reduction is to 'v' and negation, but it amounts to essentially the same thing.

** Strawson's views appear in his Introduction to Logical Theory (1952); Grice's views are as yet unpublished. Strawson's book will get one started on this issue.

Our system differs from Piaget's in that we do not accept this reduction and the consequent simplification of logic. We have not attempted this simplification in our teaching of logic, but in the tests we built to determine a person's knowledge of logic we avoided using any items that would be answered incorrectly by a person following the simplified system.

b. A second simplification which Piaget has adopted is the merging of propositions and what are called 'propositional functions'. This simplification is not one that is accepted in the field of logic. First we must explain what a propositional function is and then we shall indicate the possible significance of this merging.

Consider the statement, 'If a body floats, then it is light'. Although this might at first appear to be a standard conditional statement in sentence (or propositional) logic, note that neither of the units can stand alone. Neither of the units can meaningfully be affirmed or denied by itself, if it retains the meaning that it has in the whole. In the original statement, the words 'a body' do not refer to any particular body, so if we try to make the group, 'a body floats', stand alone, it does not make any assertion that can be called true or false. (Try to imagine how you would prove it true or false.) Since it does not say anything, it cannot stand alone.

The groups, 'a body floats' and 'it is light', are propositional functions in the original statement. They are not propositions or sentences, given the meaning they have in that original statement, because they cannot stand alone without being given a change in meaning.

Piaget in his analysis of Inhelder's floating bodies experiment, treats the statement, 'If a body floats, then it is light', as a statement in propositional logic, and is enabled to do so by merging his propositional functions and propositions.

Now why might he want to be able to treat such a statement as one of propositional logic? This is a difficult question and we can only hazard a guess, a guess that does fit in with an earlier-suggested point of noncorrespondence, his combinitorial criterion.

Note that the statement, 'If a body floats, then it is light' means roughly the same as 'All floating bodies are light', the latter clearly being a statement in class logic. Given these roughly equivalent statements, a person is free to use either one he wishes as an interpretation of the thought that all floating bodies are light. Now if for deciding whether the logic in use is propositional or class reasoning, his criterion, is, say, the combinitorial criterion, then he can adjust the form of the sentence selected to fit the decision called for by the criterion.

To be more specific, suppose that on the basis of the subject's total behavior, we decide that he is operating within a framework that includes all the possible combinations. Then we decide that the logic is propositional and we use the if-then form to represent the thought that all floating bodies are light, which he presumably is entertaining as an hypothesis. We say that he is considering the view, 'If a body floats, then it is light'.

Suppose on the other hand that the subject is not operating within a framework that considers all the possible combinations, then we judge the reasoning to be class reasoning and pick the class form of the thought that all floating bodies are light. We say that the subject is considering the view, 'All floating bodies are light'.

Perhaps Piaget did not merge propositions and propositional functions for this reason, but so merging does enable him to use the combinatorial criterion and still roughly conform to established usage of the words, 'propositional' and 'class'.

This merging of propositions and propositional functions is something we avoided in theory and tried to avoid in practice. Hence his simplification makes a difference between the sentence logic that we taught and the propositional logic that he investigated.

c. A last simplification is Piaget's apparent reduction of alethic logic (see page II-7) to ordinary propositional logic. He stretches his symbolism to cover moves in alethic logic. For example he wants to let '$p.q. \vee p.\bar{q}$' mean that each conjunction, '$p.q$' and '$p.\bar{q}$', represents a possible state of affairs. The standard symbolic logic interpretation is that at least one is true, *not* that each is possible. That Piaget also holds the standard interpretation is shown in his explanation of '$p \vee q$': "Disjunction *p* v *q* signifies that *p* is true or *q* is true or both are true." (1958, p. 296). He does not in this interpretation say that each is possible.

In his explanation of the floating bodies experiment, however, he does so interpret what he calls disjunction. Otherwise the argument that he endorses is invalid. The argument symbolized goes as follows:

Premise:

$p.q. \vee p.\bar{q}$

Conclusion:

It is false that $p \supset q$.

In the standard interpretation of the symbols, that argument is invalid. But if the premise is to be interpreted as saying that $p.q$ is possible and that $p.\bar{q}$ is possible, then it *is* false that $p \supset q$.* That is, if it is possible that p and not q, then p does not imply q, other things being equal. So Piaget has avoided the complexities of an additional symbolism and set of rules by trying to include alethic logic symbolism and rules in his basic set.

This treatment on his part is exemplified, among other places, in his analysis of the results of the floating bodies experiment. In the particular discussion we want to present he assigns the variables as follows: "*p* [is] ... the assertion that the bodies will float and *q* [is] ... any factor associated with *p* -- for example lightness (absolute)...." (1958, pp. 39-40).

---

* Assuming that 'p' and 'q' stand for propositional functions.

The statement being disproven in this selection is represented by '$p \supset q$'. Presumably this is to mean, 'If a body floats, then it is light'.*

Now with all that introduction, we hope it is clear that in this passage he is attempting to let his symbolism fit alethic logic:

> The subject may note the two possibilities combined $(p.q)\ v\ (p.\overline{q})$-- *i.e., of $p.(q\ v\ \overline{q})$ -- which constitutes the operation we may speak of as the affirmation of p independently* of the truth or falsity of q. But this operation contains $p.\overline{q}$ and amounts to discarding $p \supset q$. This is what Fran, for example, says when he declares that "the wood can be heavy (or light) and it floats"... (1958, p. 40).

The reasoning that Piaget is describing here goes like this:

> It is possible for something to float and be light; it is also possible for something to float and not be light. Therefore it is not the case that if something floats, it is light.

This argument seems legitimate, but it is not an argument in propositional logic. Arguments dealing with possibilities in this way are to be judged by the rules of alethic logic (which are not yet well worked out, by the way).

In subsuming alethic logic under propositional logic, Piaget has done some simplifying which we have not done in our presentation of logic. Instead we have avoided alethic logic.

Summary.

Thus there are points of similarity and points of difference between the logic with which we worked and the logic reported on by Piaget. Similarities are that we both seem to be dealing with logic, that we both draw a distinction between two types of logic, and that the distinction appears similar. A possible point of difference is his use of the combinitorial system criterion in drawing the distinction between class and propositional logic. A definite point of difference is the greater simplicity of the system he uses. This simplicity appears in the reduction of propositional logic to two basic operators, in the merging of propositions and propositional functions, and in the merging of alethic logic and propositional logic. We are of the opinion that our results can probably be compared with Piaget's, but want to retain as qualifications the points of non-correspondence specified above.

---

* Let us neglect the difficulties in his assignment of the variables, i.e., his merging of propositions and propositional functions, and his neglect of the difference between propositions and concepts. ('p' is here assigned by him to a proposition (or sentence), 'q' to a concept (or term)).

A further difference, not in logics, but in intensity of analysis, can be found in our attempt to study separately various principles of the types of logic with which we worked. Piaget, on the other hand, tends to speak globally about his two types of logic.

E. CHAPTER SUMMARY

In this chapter we have outlined the conception of critical thinking under which we are working, have suggested the significant role that deductive logic plays in this conception of critical thinking, have sketched out a number of types of deductive logic, have elaborated on two very basic types, and have compared these two types with Piaget's conception of similar types. Primarily in order to work within a small organization and thus maintain control of what happens, and in order to work with types of logic that are both significant and fairly well worked out, we picked class and conditional logic as the two on which we would concentrate.

Conditional logic is in our view the most important part of what we call sentence logic and what is often called propositional logic. We have judged Piaget's class logic and propositional logic similar enough to our class logic and sentence logic to declare them roughly comparable, so, with some reservations, we feel that what we have learned about the development of knowledge of logic does have a bearing on what Piaget has claimed.

CHAPTER III. Basic Data on Subjects

So that our readers will, in reading about the results, have an idea about from whom these results were secured, we are presenting basic information about the subjects before we attempt to present, discuss, and interpret the results in detail. In this chapter we shall indicate the institutional nature of the groups with whom dealt, shall comment where necessary on the instruments we used (except for the logic tests, to which we devote the entire next chapter), and then shall provide summary information about our subjects. In order that the grouping and categorizing be intelligible, we shall sketch in reasons for some of the decisions.

A. THE GROUPS WITH WHICH WE WORKED

1. The "LDT's".

At each of the grades 4-12 we selected one class-size group with which to work intensively. With each of these groups one of the members on our staff worked for one period per day (around 40-50 minutes) for four weeks (20 instructional days). Approximately three quarters of this time was devoted to the teaching of logic. The other quarter was in general devoted to advancing the subject matter that was ordinarily scheduled for that time.

Because we discovered in our pilot study that it was not possible to teach thoroughly even one type of logic in the time we had available, we decided to alternate years between the two types we selected. Class logic was taught in grades 4, 6, 8, 10, and 12. Conditional logic was taught in grades 5, 7, 9, and 11. We shall label these nine groups our 'LDT' groups, indicating groups to which Logic was Deliberately Taught.

Our LDT's were selected from an Upper New York State school system with a student population of about 8,000 in grades K-12. The district is called a 'city school district' but also comprises suburban and rural communities which were added in various centralization proceedings over the last thirty years. About one half of the students live in the city. The city itself has a population of about

14,500, plus about 14,500 college students. The main industry in this community is college education, but there are a number of medium- and small-sized factories as well.

No census figures that indicate the nature of the school district population are available, since in New York State, the boundaries of school districts do not generally coincide with political boundaries. But we can give figures that indicate some things about the occupational nature of the population of the city. For comparison purposes we are also supplying similar figures for New York State and the United States.

TABLE III-1. Percentage Occupational Make-Up of City, New York State and the United States*

Occupation	City	New York State	United States
1. Professional, Technical and Kindred Workers	25.7	12.5	10.8
2. Clerical and Kindred	18.9	18.1	14.4
3. Service workers, except private household	16.8	9.3	8.4
4. Craftsman, Foreman and Kindred Workers	7.1	12.4	13.5
5. Operatives and Kindred Workers	6.8	18.1	18.4
6. Managers, Officials and Proprietors	5.9	9.0	8.4
7. Sales Workers	5.3	7.3	7.2
8. Unemployed	3.5	5.2	4.9
9. Miscellaneous	13.5	13.3	18.8

*U.S. Bureau of the Census. U. S. Census of Population: 1960. Vol. I, Characteristics of the Population. Part 34, New York. U.S. Government Printing Office, Part I, United States Summary, Washington, D. C., 1963.

In this city there is a higher percentage of professional, technical and kindred workers than in the state and the nation.

We tried to secure classes that would typify this school system and, to the extent that this school system is typical of the United States, that would be more broadly typical. This was difficult to arrange, given the numerous pressures that impinge upon the public school administrator, but the people with whom we worked were very helpful and did the best they could.

Our classes in grades 4-6 were from an elementary school which contained a broad distribution of backgrounds. The classes were existing units which were turned over to our staff members for the designated period.

In grades 7-8, the classes made available to us were arithmetic, and the 9th grade class composed of students supposedly with sufficient ability to be taking algebra, but who were not doing so for one reason or another.

Because it was not possible to take so much time from the course of instruction in high school classes of average or above-average ability (the New York State Regents examinations were a factor operating here), our classes at the high school grades were composed of volunteers from study halls.

2. The "LNDT-1's".

At each grade level from the same school system we requested a fairly comparable class in which Logic was Not Deliberately Taught. Hence the designation, 'LNDT'. Since there was another set of students to which logic was not directly taught, we added the numeral '1' to the designation. The LNDT-1's served two purposes: to provide more subjects for our study of the natural-cultural development of knowledge of logic, and to provide a check on the efficacy of our teaching. All the data that we have on the LDT's we also have on the LNDT-1's.

3. The LNDT-2's.

Another group of students, the LNDT-2's, to whom we did not teach logic was selected from a nearby school system. It is in an area that is part of the larger community of which the city is the hub, and, although it is a rural community, it is a suburb of the city.

We did not gather the same data on the LNDT-2's as on the others. A socioeconomic status index was not secured, and the IQ scores were not from the same test. These subjects were used primarily for data on the natural-cultural development of knowledge of logic. We attempted to test all the students in grades 4-12 in the school system with our logic tests.

B. THE SOURCES OF OUR DATA.

We have the following data on our subjects:

a. Grade level in school.
b. Chronological age.
c. Sex.
d. I.Q. score.
e. An index of socio-economic status (except LNDT-2's).
f. Pre-test and post-test scores on the appropriate Cornell Deduction Test.

In this section we shall discuss, where appropriate, the instruments and/or techniques we used to gather these data. However, we shall not here be concerned in any detail with the instruments that we developed, the conditional and class reasoning tests, since a separate chapter is devoted to them.

1. Readily Obtainable Information.

The first three items, grade level, chronological age, and sex are routinely available from school records. Chronological age was determined as of the date on which the pre-test was given, which is approximately the date on which instruction at the given grade level started for the LDT's of that grade level. This occurred in February or March of 1964, depending on the schedule for that particular grade.

I.Q. score, although on the school records, required more discretion in data gathering. For the LDT's and LNDT-1's, which were from the same school system, we chose the latest available score on the Lorge-Thorndike Intelligence Tests. Where both verbal and non-verbal scores were available, we took the arithmetic mean; where only one was available we used that. Fourth graders were tested with level 2 Lorge Thorndike, fifth and sixth graders with level 3, and the rest with level 4. All I.Q. testing had been done within the previous two years except for that of the eleventh and twelfth graders, most of whom were tested in sixth grade and eighth grade respectively. Thus we were able to secure fairly comparable scores from the school's records for the LDT's and LNDT-1's.

The use that we made of the LNDT-2's did not require the same I.Q. test. The school system from which they were drawn used a wide assortment of tests. The majority of the scores of LNDT-2's are on the California Test of Mental Maturity, 57S, but because many are from other tests, the mean IQ's which we report should be understood to be rough estimates.

2. Less Readily Obtainable Information.

a. Socio-economic status. Because of our interest in the relationship between socio-economic status (henceforth called 'SES') and knowledge of logic, we made a rudimentary attempt to secure for each LDT and LNDT-1 a number that bears a relation to SES. This number is based upon the occupation of the parent, making use of Warner's seven-place occupational scale (1949, pp. 140-41). Occupations with the highest socio-economic status receive a rating of 1, and those with the lowest socio-economic status receive a rating of 7.

Two raters separately applied Warner's seven-point scale to the occupation listed in the school records for each student's father, or mother if she was the principal means of support. When the occupation did not appear in the school records, we made an effort, which was generally successful, to obtain it. We did not, however, approach the subject's parents and ask them, feeling that this approach would be resented, unless we devoted more time to it than we had available. In exactly 11 cases out of 428 (the total number of LDT's and LNDT-1's), we were unable to determine the occupation with enough specificity to make an assignment. In those cases we made an arbitrary assignment of 6, because it has been our experience that lack of information or information that is too vague is a good indicator of low socio-economic status. Naturally this assumption does not hold in every case, so we made every effort to keep our use of it to a minimum.

We made the following additional amendments and/or additions to the Warner scheme as presented on pp. 140-41 of Social Class in America. These changes were made on the basis of our knowledge of the prestige structure of this community, with which each rater has had considerable experience.

1. Unemployed people are not given a number in the Warner scheme. Although inevitably there will be some errors in so doing, we assigned them the number 7. At this time in the economic life of our country, this assignment is more realistic than it would have been, say, during the Depression.

2. People in the armed forces are not given a number in the Warner scheme. We assigned the number 3 to officers (rank unspecified) and 6 to enlisted men.

3. College teachers also are not assigned a number. To them we assigned the number 1.

4. Warner assigned the number 5 to dime store clerks. We changed that to 6.

5. Warner assigned the number 5 to hardware salesmen. We changed that to 4.

6. As we interpret his chart, there was no number assigned to qualified electricians who do not own their own businesses. We assigned the number 5, since he assigned this number to carpenters and plumbers who do not own their own businesses.

After each rater had practice-rated a class from a previous study, the raters discussed their differences of opinion, which were small in magnitude and number, settled upon the above list of adjustments to the Warner scale, and proceeded independently to rate all 428 LDT's and LNDT-1's. For purposes of estimating reliability of our ratings, we computed a Pearson product-moment correlation coefficient between the two sets of ratings. It was .95, indicating high scorer reliability.

Since we were not able to use both sets of ratings, we adopted the simple procedure of flipping a coin to select one set, which was used.

b. The Cornell Deduction Tests. There were two Cornell Deduction Tests, "The Cornell Class-Reasoning Test, Form X," and "The Cornell Conditional-Reasoning Test, Form X." The former was administered as a pre-test immediately before teaching the LDT's for the grade, and as a post-test six weeks after the completion of teaching to all students in grades 4, 6, 8, 10, and 12; the latter was administered similarly to all students in grades 5, 7, 9, and 11.

In this chapter we shall report only total scores, which were computed making use of a correction formula. The score reported is rights minus $\frac{1}{2}$ wrongs plus 27 (R- W/2 + 27). Each test had 72 items making the possible range run from minus 9 to plus 99. This scoring procedure and many other features of the test will be discussed in the next chapter. Other scores and their analyses will be presented in other chapters.

That completes the presentation of the sources of data, except of course for the discussion of the two logic tests which will follow in the next chapter.

C. THE FIGURES.

Given the previous descriptions of the groups and methods and/or instruments for securing data, the following table provides a general picture of the sort of subjects with whom we worked. Altogether there were 803 subjects, 217 LDT's, 211 LNDT-1's, and 375 LNDT-2's. Mean IQ for all subjects is roughly 114* and mean SES rating for LDT's and LNDT-1's combined is 3.7. Thus our subjects are above average, as one might expect for a college community in the Northeast.

---

*We say "roughly" because purity of IQ tests is not achieved because of the LNDT-2's.

TABLE III-3. Basic Information on Our Subjects

		N			Chronological Age(mos.)		IQ		SES		Class Reasoning				Conditional Reasoning			
											Pre-test		Post-test		Pre-test		Post-test	
Grade	Group	Male	Female	Total	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
04	LDT	13	12	25	116.6	5.0	109.4	18.3	4.1	1.9	46.4	14.3	45.8	16.0				
	LNDT-1	11	11	22	119.0	5.0	109.0	19.1	3.1	2.3	44.3	14.9	46.2	17.5				
	LNDT-2	19	28	47	117.2	5.9	108.3	14.2			43.4	16.3	47.2	15.1				
05	LDT	13	14	27	128.2	4.4	111.6	16.0	3.6	1.9					40.5	9.7	45.2	13.6
	LNDT-1	14	12	26	130.5	6.5	111.5	19.8	4.0	2.0					43.0	13.0	46.4	14.8
	LNDT-2	26	23	49	128.7	6.7	105.1	11.9							42.9	10.5	47.0	9.5
06	LDT	14	11	25	141.1	3.8	114.0	13.4	4.6	2.1	55.4	11.2	64.9	13.7				
	LNDT-1	23	11	34	141.4	4.0	110.5	14.1	3.8	2.1	54.3	16.9	56.8	17.1				
	LNDT-2	18	26	44	142.6	5.8	113.6	15.6			52.6	13.3	56.0	14.8				
07	LDT	14	10	24	152.0	4.5	122.2	7.9	3.1	1.9					52.6	10.6	55.0	10.3
	LNDT-1	11	14	25	152.4	4.8	118.7	8.3	2.8	1.7					54.1	10.7	56.6	10.7
	LNDT-2	23	27	50	153.7	5.4	113.7	13.5							49.8	13.0	53.6	12.8
08	LDT	11	16	27	163.5	3.9	120.4	6.9	3.9	1.9	67.7	11.5	71.5	13.9				
	LNDT-1	11	13	24	164.2	6.5	113.9	10.1	3.6	2.5	58.8	12.2	61.6	13.3				
	LNDT-2	28	21	49	169.2	7.3	108.3	16.5			52.8	20.0	58.6	16.5				
09	LDT	11	06	17	183.4	9.0	110.0	13.5	4.4	1.8					48.9	9.0	46.3	10.9
	LNDT-1	12	11	23	179.0	6.4	109.4	13.8	4.2	1.9					50.8	11.6	55.0	13.1
	LNDT-2	26	14	40	182.3	8.1	104.9	16.6							52.3	11.3	55.1	10.6
10	LDT	15	07	22	190.7	6.0	119.7	10.6	3.7	2.0	69.6	11.7	77.9	12.5				
	LNDT-1	08	10	18	189.3	5.2	131.5	7.7	2.7	1.5	77.2	8.6	80.1	9.2				
	LNDT-2	18	17	35	191.1	8.4	114.3	15.7			68.5	12.7	64.1	19.2				
11	LDT	14	12	26	199.8	4.3	116.0	11.9	3.5	1.9					63.7	10.5	80.7	16.4
	LNDT-1	11	11	22	204.0	7.2	108.1	11.6	4.2	2.0					51.9	16.3	58.5	11.6
	LNDT-2	19	11	30	206.5	6.2	102.1	13.6							53.5	12.9	56.4	11.7

TABLE III-3. Basic Information on Our Subjects - cont.

					Chronological Age(mos.)		IQ		SES		Class Reasoning Pre-test		Class Reasoning Post-test		Conditional Reasoning Pre-test		Conditional Reasoning Post-test	
Grade	Group	Male	Female	Total	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
12	LDT	12	12	24	210.8	4.0	127.8	16.9	3.1	1.9	80.4	12.8	87.2	8.0				
	LNDT-1	08	09	17	212.2	2.9	134.2	11.4	3.0	1.5	81.9	8.1	83.5	15.6				
	LNDT-2	19	12	31	218.5	9.5	105.2	17.6	3.0		62.6	15.3	62.0	18.6				
Total	LDT	117	100	217			116.8		3.8									
	LNDT-1	109	102	211			116.3		3.5									
	LNDT-2	196	179	375			108.4											
Grand Total		422	381	803			113.8		3.7									

CHAPTER IV. The Cornell Deduction Tests

In order to measure our subject's knowledge of logic, we found it necessary to develop our own instruments, there being no instruments available that suited our purposes. Most existing instruments which are partly or wholly aimed at testing for knowledge of logic do so on a rather global scale, and do not attempt to secure separate measures of basic principles of logic. A key feature of our approach is our interest in specific principles of logic on the assumption that principles vary in the ease with which, and the level on which, they are acquired.

Two tests which do attempt separate interpretation for specific principles of logic and which made at least some attempt at comprehensiveness are worthy of mention here. Unfortunately neither of these were usable, for reasons which we shall give.

In 1919 Cyril Burt published an instrument called "Graded Reasoning Tests" for use with individuals as opposed to groups. He did in developing the instrument attempt to make a comprehensive coverage of basic principles of certain types of logic. This instrument provided Burt with data which he used to support statements about when children develop certain logical abilities (Burt, 1919). However we were not able to use his instrument for several reasons:

1. Time limitations required the use of a group test. Possibly Burt's instrument is adaptable (Fairgrieve, 1921), but we did not try because of other difficulties with it, given our purposes.

2. It calls on the person being tested to invent and supply a premise (sometimes called a "suppressed premise"). The trouble here is that any argument can be made valid by supplying some premise. Unless the individual is tested individually it is difficult to see how to find out just what premise he was adding.

3. The final product was held by Burt to provide an overall score. There is no score possible on a type or sub-type.

4. Often a given form of argument is tested for by only one or two items.

Granted that with individual testing, the examiner can by asking the right questions often get a fairly good idea of the nature of a person's difficulty with an item, but there is danger that something else will without detection affect the response.

In a Ph.D. thesis completed in 1961, Shirley Hill reported on an instrument that she developed in order to investigate the degree of knowledge of logic among 6, 7, and 8 year olds. Her test appears to be satisfactory for administration to groups, and is based on an attempt at a comprehensive coverage of basic principles of valid arguments in class and sentence logic, but it was still not satisfactory for our purposes.

The test was built so that the most important feature of the distinction between a valid and invalid argument could not be tested for. All proposed conclusions either followed necessarily or contradicted the premises.

Although arguments in which the conclusion contradicts the premises are a sub-class of invalid arguments, a more important sub-class is the group of arguments in which the conclusion does not follow, but also in which the conclusion does not contradict the premises. People are rarely trapped into thinking that an argument is valid in which the conclusion actually contradicts the premises. The important distinction is between a valid argument and between one which someone might be inclined to call valid, but which really is not valid. The mastery of this distinction is not tested for in this test. Thus it too was unsatisfactory for our purposes.

In this chapter we shall describe the features that we deliberately built into our instruments, and the rationale for having these features; we shall present information bearing on their reliability and validity; and lastly we shall suggest some lines along which we feel further development is possible.

A. THE STRUCTURE OF THE TESTS

The two tests that we developed are called "The Cornell Class-Reasoning Test, Form X" and "The Cornell Conditional-Reasoning Test, Form X". They may be found in the Appendix to this report. Each is a 72-item multiple-choice test designed for use with any of the grades with which we were concerned (grades 4-12). Each item in virtue of its logical form was expected to play a role in measuring a person's knowledge of some principle or combination of principles; in virtue of its content it was expected to contribute to one of the three content components that we built into the test. First we shall discuss the logical form of the items; then we shall discuss their content.

1. The Logical Form of the Items.

Each test was constructed in such a manner that it tested for knowledge of twelve principles or combinations of principles. Six items were assigned to each principle or combination of principles. Henceforth we shall refer to each set of six such items as an 'item group'. There are therefore twelve item groups in each test.

In the conditional reasoning test the list of principles or combinations embodied by the items corresponds very closely to the list of basic principles of conditional reasoning given in Chapter II. In order that we could have some combinations of principles we left the twelfth one off, on the assumption that it is less often needed than the others; and used the eleventh one only in a combination with another. Principles One through Ten, however, each had item groups exclusively assigned to them. Table IV-1 gives the selection and extent of combination of principles of conditional reasoning.

In the class reasoning test each of the basic principles had an item group assigned to it exclusively, except Principle Three which had two item groups assigned to it. Item groups #3 and #4 embody symbolized arguments that are different enough to justify being distinguished, though they are both covered by Principle Three. Thus nine item groups are assigned to specific principles of class logic. The other three item groups are assigned to combinations of the principles or to double applications of the principles. See Table IV-2 for the specific nature of the assignment.

TABLE IV-1. Logical Form of and Answers to Items in "The Cornell Conditional Reasoning Test, Form X"

Item Group	Princi-ple(s)	Basic Form, which was used for the first two concrete familiar items (CF1 and CF2), the symbolic item (SY) and the suggestive item (SU)	Answer for Basic Form	Form of CF3	Answer to CF3	Form of CF4 (Same answer as basic form)	Item Numbers CF1*	CF2	CF3	CF4	SY*	SU*
1	1	If p, then q p : q	Yes	If p, then q p : not q	No	not p If not p, then q : q	7	40	27	14	19	31
2	2	If p, then q not p : not q	Maybe	Same as basic form	Maybe	p If not p, then q : not q	9	13	26	18	34	23
3	3	If p, then q q : p	Maybe	Same as basic form	Maybe	q If not p, then q : not p	11	24	32	37	30	41
4	4	If p, then q not q : p	No	If p, then q not q : not p	Yes	not q If not p, then q : not p	8	35	29	16	22	39
5	5	If p, then q If q, then r : if p, then r	Yes	Same as basic form	Yes	If q, then r If not p, then q : if not p, then r	45	55	66	52	49	73
6	6	If p, then q : if not q, then not p	Yes	Same as basic form	Yes	If not p, then q : if not q, then p	46	69	74	56	61	50
7	7	If p, then q : if q, then p	Maybe	Same as basic form	Maybe	If not p, then q If q, then not p	44	57	77	70	59	64
8	8	p only if q not q : p	No	p only if q not q : not p	Yes	not q not p only if q : not p	12	21	42	25	15	36
9	9	p only if q p : q	Yes	p only if q p : not q	No	not p not p only if q : q	10	17	20	33	38	28

TABLE IV-1 cont.

Item Group	Principle(s)	Basic Form, which was used for the first two concrete familiar items (CF1 and CF2), the symbolic item (SY) and the suggestive item (SU)	Answer for Basic Form	Form of CF3	Answer to CF3	Form of CF4 (Same answer as basic form)	Item Numbers					
							CF1	CF2	CF3	CF4	SY	SU
10	1&5 or 1 applied twice	If p, then q p If q, then r : not r	No	If p, then q p If q, then r : r	Yes	If not p, then q If q, then r not p : not r	43	51	62	72	67	76
11	10	p, if and only if q not p : q	No	p, if and only if q not p : not q	Yes	p not p, if and only if : q	47	54	63	58	78	60
12	1&11	If p, then q p r only if q : r	Maybe	Same as basic form	Maybe	If not p, then q r only if q not p : r	48	53	71	68	65	75

Note: The symbol ':' is used to introduce the proposed statement.

* 'CF' stands for 'concrete familiar', 'SY' for 'symbolic', and 'SU' for 'suggestive'.

TABLE IV-2. Logical Form of and Answers to Items in "The Cornell Class-Reasoning Test, Form X"

						Item Numbers					
Item Group	Princi-ple(s)	Basic Form, which was used for the first two concrete familiar items (CF1 and CF2), the symbolic item (SY) and the suggestive item (SU)	Answer for Basic Form	Form of CF3 (same answer as basic form)	Form of CF4 (same answer as basic form)	CF1*	CF2	CF3	CF4	SY*	SU*
1	1	All As are Bs. : At least some As are not bs.	No	X is an A : X is not an A.	At least some As are Bs. : no As are Bs.	8	35	29	16	22	39
2	2	All As are Bs. All Bs are Cs : All As are Cs.	Yes	All As are Bs. X is an A. : X is a B.	At least some AS are Bs. All Bs are Cs. : At least some As are Cs.	7	40	27	14	19	31
3	3	All As are Bs. : All Bs are As.	Maybe	All As are Bs. X is a B. : X is an A.	At least some As are Bs. : All Bs are As.	11	24	32	37	30	41
4	3	All As are Bs. All Cs are Bs. : At least some Cs are As.	Maybe	All As are Bs. X is a B. : X is an A.	At least some As are Bs. All Cs are Bs. : At least some As are Cs.	9	13	26	18	34	23
5	4	No As are Bs. : No Bs are As.	Yes	No As are Bs. X is an A. : X is not a B.	No As are Bs, and there are Bs. : At least some Bs are not As.	10	17	20	33	38	28
6	5	All As are Bs. No Cs are Bs. : At least some As are Cs.	No	All As are Bs. X is not a B. : X is an A.	At least some As are Bs. No Cs are Bs. : All As are Cs.	12	21	42	25	15	36
7	6	All As are Bs. No Cs are As. : No Cs are Bs.	Maybe	All As are Bs. X is not an A. : X is not a B.	All As are Bs. No Cs are As. : At least some Cs are not Bs.	44	57	77	70	59	64

TABLE IV-2 cont.

Item Group	Princi-ple(s)	Basic Form, which was used for the first two concrete familiar items (CF1 and CF2), the symbolic item (SY) and the suggestive item (SU)	Answer for Basic Form	Form of CF3 (same answer as basic form)	Form of CF4 (same answer as basic form)	CF1*	CF2	CF3	CF4	SY*	SU*
8	7	No As are Bs. No Cs are Bs. : At least some As are Cs.	Maybe	No As are Bs. X is not a B : X is an A.	At least some As are not Bs. At least some Cs are not Bs. : At least some As are Cs.	48	53	71	68	65	75
9	8	All As are B. : All non-Bs are also non-As.	Yes	All As are Bs. X is a non-B. : X is a non-A.	All As are Bs, and there are non-Bs. : At least some non-Bs are non-As.	45	55	66	52	49	73
10	1 & 8	All As are Bs. All non-Cs are also non-Bs. : At least some As are not Cs.	No	All non-As are non-Bs. X is a B. : X is not an A.	At least some As are Bs. All non-Cs are non-Bs. : No As are Cs.	43	51	62	72	67	76
11	2 applied twice	All As are Bs. All Cs are Ds. All Bs are Cs. : At least some As are not Bs.	No	All Bs are Cs. All As are Bs. X is an A. : X is not a C.	At least some As are Bs. All Cs are Ds. All Bs are Cs. : No As are Ds.	47	54	63	58	78	60
12	2 & 8 or 8 applied twice or 2,4&5	All Bs are Cs. No Ds are Cs. All As are Bs. : No Ds are As.	Yes	All Bs are Cs. X is not a C. All As are Bs. : X is not an A.	All Bs are Cs. No Ds are Cs. At least some As are Bs. : At least some As are not Ds.	46	69	74	56	61	50

Note: The symbol ':' is used to introduce the proposed statement.

* 'CF' stands for 'concrete familiar', 'SY' for 'symbolic', and 'SU' for 'suggestive'.

An additional logical principle which is often invoked in both tests is the principle of double negation: that two negatives make a positive. Explicitly the principle goes as follows: "If it is false that a statement is false, then the statement is true, and if a statement is true, then it is false that it is false." Now there are qualifications to be added to this principle, but this simple statement of it holds in the cases in which it is needed. We do not consider this principle to be tested for in this test, but rather assume that it is mastered. To the extent that our assumption is false, the test is also a test for this principle.

2. The Three Content Components.

In a frequently-mentioned study Wilkins (1928) specified four types of content and attempted to see which kind made syllogisms more difficult. We have selected her first, second, and fourth types for our tests. Her four types are:

a. Concrete familiar, in which the content mentioned is concrete articles and qualities with which the subject has been associated. However, no statements are made which the subject has reason to believe to be true or false, because the specific objects referred to are not known by the subject. For example instead of saying, "All cats are black", a statement with which the subject is likely to disagree, a statement might be, "Mary's cats are black". The latter is less likely to meet with agreement or disagreement, although there is still the possibility of disagreement, if the subject identifies the Mary of the statement with some Mary he knows.

b. Symbolic, in which symbols like 'x', 'y', 'A', and 'B' are used in key places, instead of terms that refer to particular objects. An example of a sentence using symbolic content is the following: 'All A's are B's.'

c. Unfamiliar, in which the basic terms are scientific terms with which the subject is expected to be unfamiliar, or are "nonsense words invented to sound like scientific terms" (p. 13). For example, 'All hentras are globiculous', is a sentence with unfamiliar content.

d. Suggestive, in which the content is familiar, but the truth status of this content is known by the subject. Furthermore, "the truth or falsity of these statements [conclusions] was at variance with their validity." (p. 13). In other words the truth status of the conclusion was different from the validity status of the argument. A person might be tempted to mark an argument valid because he agrees with the conclusion, or invalid because he disagrees with it.

In our tests the three of Wilkins' four content components that we used are the ones we feel a person is most likely to encounter in realistic reasoning situations in which his reasoning ability will make a difference. Our three content components are the concrete familiar, the symbolic, and the suggestive. We do not think that what she calls 'unfamiliar' content is likely to be encountered in situations that make a difference. If a person meets arguments with such content, he will generally not know what to do with the conclusion anyway.

This is not to say that arguments with such content are useless in teaching, for they can be very helpful in isolating questions of form, and they can be useful in arousing interest, if handled properly. Furthermore it might well be that they are good indicators of grasp of form. But ultimately our interest is a practical one, so we, given the testing time limits forced upon us by the situation, did not include this type of content in our tests.

The six items in each item group were assigned to the three content components in the following way. Four of the items had concrete familiar content, a fifth had symbolic content, and a sixth had suggestive content. There were some formal variations among these, incidentally, which we shall describe next.

A basic logical symbolic form was assigned to each item group. It appears in Column 3 in Table IV-1 and Table IV-2. This basic form was used for two of the concrete familiar items, the symbolic item, and the suggestive item. The third and fourth concrete familiar items embodied slight formal changes, in order to provide greater variety in the representatives of each principle or combination.

Because class and conditional reasoning differ in their basic structure, these variations differed.

In the conditional reasoning test the third concrete familiar item negated the conclusion, unless there were good reasons not to do so. That is, if the conclusion called for by the basic symbolic form is positive, the one provided is negative; and if the conclusion called for is negative, then the one provided is positive; unelss, as we said, there are good reasons not to do so.

What are the reasons? There are two:

1. If the basic form is invalid, then denying the conclusion reduces the temptation to judge it valid, making the item considerably easier. So for those four item groups in which the basic form is invalid, the third concrete familiar item conforms to the basic form.

2. If the conclusion is itself a conditional statement, a denial of it becomes awkward to state. Note that the denial of "If Sam misses the bus, then he will walk to school" is not "If Sam misses the bus, he will not walk to school", which is not awkward to state. Instead the denial is, "It is not the case that if Sam misses the bus, he will walk to school", which is awkward. Hence the third concrete familiar items conformed to the basic logical form in two additional cases (Groups 5 and 6).

The fourth concrete familiar item in conditional reasoning varied from the basic form by having one of the basic sentences negated throughout, and by having premises in different order, when there are two or more premises. Table IV-2 presents symbolic representation of this variation.

In class reasoning the third concrete familiar item contained at least one class membership statement in the premises instead of all class inclusion and exclusion. A class membership statement asserts the membership or nonmembership of one thing in a class, whereas a class inclusion or exclusion statement speaks about the relation between classes.

The fourth concrete familiar item in the class reasoning test contained at least one statement of partial inclusion.

The composition of each item group is presented in Tables IV-1 and IV-2.

3. Item Format.

We sought an item format that would meet the following criteria:

a. Provide a way for a subject to show that he knows that a proposed conclusion does not follow, even though it does not contradict the premises.

b. Avoid technical language, yet ask whether an argument is valid.

c. Provide more than two alternatives in multiple choice form.

d. Provide fairly equal numbers of each type of answer.

e. Allow for separation of different items of the same logical form.

f. Be understandable to fourth graders without insulting twelfth graders.

g. Avoid confusing truth and validity.

h. Not require elaborate directions.

i. Not require an elaborate scoring procedure.

j. Minimize irrelevant errors.

After much experimentation and discussion we settled upon a format which asks the subject to suppose (a) certain premise(s), and to decide whether, on the basis of the supposition, a further thing would be true. We shall call this further thing the 'proposed statement' and the original supposition the 'supposed statement(s). There are three possible answers, 'Yes', 'No', and 'Maybe', each of which is explained in the directions by means of six sample problems. Furthermore a brief explanation of each response appears at the top of every page.

The response 'Yes' indicates that the subject thinks the proposed statement follows necessarily. 'No' signifies that the proposed statement contradicts what has gone before. 'Maybe' means that the proposed statement neither follows necessarily nor contradicts -- that its truth is not necessarily determined by the establishment of the truth of the premises.

We introduced the 'No' response partly in order to make possible three choices instead of the two suggested by the distinction between validity and invalidity. We wanted three choices in order to lessen the attractiveness of wild guessing and to lessen the chances of its success. We were able to introduce this 'No' response as a correct answer by taking conclusions to valid arguments and then denying them. For the 'No' response, the proposed statement is then a denial of a conclusion that follows necessarily from the supposed statement(s). A second reason for having a 'No' response is that doing so enabled us to avoid a preponderance of 'Yes' answers over 'Maybe' answers.

In introducing this 'No' response we were well aware that we were running a risk. There is an additional principle involved in each of these items: That the denial of a statement which follows necessarily from supposed statements can not be accepted, if the supposition is accepted. This principle is elementary, but it must be invoked. Failure to invoke it, as contrasted with failure to know the basic principle(s), could explain errors that are made. Thus there is an additional possibility of error of measurement. We feel that the advantages gained are worth the risk.

The reader is referred to the copies of the tests in the Appendix to see the sample problems and initial directions. Let it suffice here to present the top-of-the-page interpretation of the answers and an item from the class reasoning test:

Here is a reminder of the meaning of the possible answers:

A. YES It must be true.
B. NO It can't be true.
C. MAYBE It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

---

7. Suppose you know that

All the cars in the garage are Mr. Smith's.
All Mr. Smith's cars are Fords.

Then would this be true?

All of the cars in the garage are Fords.

A. YES
B. NO
C. MAYBE

In order to avoid possible confusion, separate answer sheets were not used. Subjects circled an answer on the right.

Although there are many strong arguments on both sides we directed students not to guess wildly and used a scoring formula. We did this because, in addition to the standard arguments, we feel that a critical thinking test should not encourage wild guessing, and that part of being a critical thinker is knowing when you do not know.

The following direction appears on the front of both tests:

> DO NOT GUESS WILDLY. There is a scoring penalty for guessing wrong.
> If you think you have the answer, but are not sure, mark that answer.
> But if you have no idea, then skip the question.

The scoring formula which we used is: Score = R - W/2 + 27. The 27 was added in order to make negative scores unlikely.

4. Item Arrangement.

The first six items are sample items in each test. They are drawn from other types of logic than class and conditional reasoning. Items 7 through 78 in each test are divided in two parts, although the subject is not informed of this fact. The first part contains the item groups for the six easiest forms, as determined in our tryouts. The second part contains the item groups for the more difficult forms. Thus it might be possible to cut the test in half for testing younger children or for other purposes.

Since our item groups are generally numbered in recommended teaching order, and not necessarily in order of increasing test difficulty, some of the groups in the second half of the conditional test represent principles in the first six on the teaching order list.

Items were arranged so that no two items from the same item group appeared on the same page, and so that the distribution of answers would appear fairly random to the subjects. The specific assignment of items to test item numbers is shown in Tables IV-1 and IV-2.

B. OPERATIONAL DEFINITIONS OF 'MASTERY OF X PRINCIPLE'

Our basic interest is in determining whether or not a given principle of logic has been mastered. Since there are no established criteria for making such a determination, we developed our own, realizing of course that our results are correct only to the extent that our criteria are satisfactory and that the question of the satisfactoriness of our criteria is not strictly an empirical matter. It is partly also a matter of the meaning of the term 'mastery' and of the meaning of the principle involved.

Since there is no outside criterion, one principal empirical question is whether the criterion operates reliably (or whether it is based upon a reliable measure). Another empirical question might be whether the criterion-based judgments are in agreement with the judgments of experts, but of course their judgments must be based upon the non-empirical meaning features mentioned above. Since these judgments about meaning require a great deal of attention to the operation of the particular criterion and there is a limit to the amount of expertise that can be exercised, we did not go outside our own staff for such expert judgments. (We did seek outside opinion about the validity of the total tests and will present that later.)

What a person interested in our research must do is to examine our criteria in very specific terms and decide for himself whether they reasonably conform to the meaning of the phrase, 'mastery of x principle'. In short the non-empirical aspects of our criteria are not arbitrary, but they are matters of intelligent judgment rather than empirical investigation.

Our criteria took the form of operational definitions. The operational definition structure that we used is as follows (abstracted from Ennis, 1964c): First there is a specification of an operation performable by an investigator; this specification appears in a conditional clause. Next comes a statement of some sort of relationship between the making of specifiable observations and the application of the term being defined. In effect the operational definition gives an incomplete empirical interpretation of the meaning of an abstract term, because the relationship is claimed to hold when the operation has been performed, but no claim is made about the status of the relationship at other times, or when other operations have been performed.

Furthermore the operational definition should have some explicit qualifiers like 'probably' and 'under standard conditions' if there is a reasonable chance that something might go wrong in the measuring process. Since there ordinarily is such a chance in dealing with human beings, it is best to include such qualifiers in our operational definitions.

First we shall give an informal presentation of one of our operational definitions; then we shall give the same definition formally; finally we shall discuss some dangers in our approach.

Put informally the operational definitions of mastery of one of our principles go as follows: Given that we have administered the "Cornell Conditional Reasoning Test, Form X" under standard conditions and have scored it according to the key given in Table IV-1. Then if a subject gets right at least five of items 8, 16, 22, 29, 35, and 39, we can be fairly sure that he has mastered Principle #4 (Given an if-then sentence, the denial of the then-part implies the denial of the if-part). Furthermore if he gets fewer than four of those items right, we can be fairly sure that he has not mastered Principle #4.

One might choose to regard the foregoing as two operational definitions or as one. It does not matter. For convenience we regard it as two operational definitions. The distinction between them appears in the following pair of formal operational definitions of the same principle. The first gives a sufficient condition of mastery, given the operation; the second gives a necessary condition of mastery, given the operation:

1. If Y is given "The Cornell Conditional Reasoning Test, Form X" under standard conditions; then if Y answers correctly at least five of items 8, 16, 22, 29, 35, and 39, Y has probably mastered Principle #4.

2. If Y is given "The Cornell Conditional Reasoning Test, Form X" under standard conditions; then it is probable that Y has mastered Principle #4 only if Y answers correctly at least four of these items: 8, 16, 22, 29, 35, and 39.

These two operational definitions of 'mastery of Principle #4' served as our criteria for estimating the per cent of students at each grade level who had and had not mastered Principle #4 of conditional reasoning. We used similar operational definitions for each of the other principles for which we had a corresponding item group.

It is important to note that the above pair of operational definitions does not specify a decision for a subject who correctly answers exactly four of the items in an item group. If a subject correctly answers at least five, then he is judged to have mastered the principle. If he does not correctly answer at least four (that is, if he correctly answers three or fewer), then he is judged not to have mastered the principle. But the person who has a score of four is not put in either category.

There is a reason for framing the operational definitions this way. Such people we think are borderline cases, so we have accommodated the operational definitions to this fact. We do not desire to be committed, even with the qualifications that exist in the definition, when a person answers correctly exactly four of an item group. We do not want to say that he has mastered the principle, and we do not want to say that he has not mastered the principle. We want to leave the question open.

One of the dangers in this operational specification of the meaning of 'mastery of X principle' is that we might be taken to have exhausted the meaning of the phrase with our necessary and sufficient conditions; that is, we might be taken to have equated the meaning of this phrase with a finite set of "behaviors", as people so often say. A careful examination of the logical form of our definitions should make it clear that we are not doing this. If we were to do so, then we would be committed to saying that no other test of mastery of these principles can attain validity simply by virtue of the meaning of 'mastery of X principle'.

Furthermore our qualifying terms, 'probably' and 'under standard conditions' protect us from being unalterably committed to the view that a subject who by wild guessing answers correctly five of the six items in a group knows the principle. A behavioristic interpretation would commit one to such a view. Our judging procedure will classify such a person as one who has mastered the principle, but the looseness of our definition enables us to avoid such a commitment.

In summary, even though operationism is often associated with behaviorism, we want it to be clear that our operational definitions should not be so associated.

A second danger in our approach arises from the number of items in an item group: six. The use of six items instead of more represents a compromise that we struck. Pressures that operated to keep down the number of items were the need to avoid taking too much school time of our subjects, and the desirability of including a variety of principles and combinations thereof. Given these pressures, together with the fact that our items are so directly and clearly related to their principles, a study of our tryout test reliabilities made us willing to utilize part scores based upon six items. This use of these part scores was fairly successful, from the point of view of reliability. For a discussion of the reliability of parts and wholes at each grade level, see the next section.

A third danger in our use of these operational definitions is the possible confusion with respect to those item groups which represent combinations of principles and with respect to Item Groups #3 and #4 of the class reasoning test, both of which embody Principle #3.

We shall deal with the latter problem first. Since operational definitions do not present complete interpretations of the meaning of terms, the double pair of operational definitions of 'mastery of Principle #3' (class reasoning) supplement each other. Does this imply that we must have complete agreement between these two item groups? No, it does not. Systematic and random variations, errors, and misunderstandings are bound to slip in. One does not even expect (nor achieve) complete agreement among thermometers of different types, although they are supposedly measures of the same thing, temperature, and are mentioned in operational definitions of 'temperature'.

A glance of the difficulty index development on class reasoning Item Groups #3 and #4 (Table IV-9, p. 31) shows that these two item groups are not vastly different from each other, although Item Group #4 appears more difficult, probably because of its greater complexity.

The other problem, that of confusion resulting from the combination of principles for some item groups, may be stated in two parts:

1. The mastery of which principle is being defined?
2. When alternative combinations of principles can be used to justify a correct judgment for a given item group, which of the alternatives is being defined?

The first part of the problem can be answered by stating that it is 'mastery of the combination of Principles X and Y' that is being defined.

The second part of the problem is not one we have been able to solve to our satisfaction. It is probably the result of the redundance, mentioned in Chapter II, that is built into the list of principles. This redundance is the cost of intelligibility and utility for teaching.

One way out is to say that the necessary condition operational definition can be used to show, if the necessary condition of at least four correct is not met, that probably no one of the combinations has been mastered. Furthermore the sufficient condition operational definition can be used to show if the sufficient condition definition of at least five right has been met, then at least one of the combinations has probably been mastered. Thus the test results have a verbal interpretation, but the test is not serving, in these sufficient condition instances, as a vehicle of interpretation of particular principles or combinations thereof.

These operational interpretations represent a new approach to the relation between empirical data and the concept of mastery. We would like to see it tried and tested in other areas, and are using it because we feel it has promise.

C. RELIABILITY

Because of the principle and content components of the tests, it did not seem appropriate to compute a split-half reliability estimate.* There just was no way to divide the tests into two equal halves. So we are reporting a test-retest reliability estimate based upon the groups that did not receive logic instruction, the LNDT-1's and the LNDT-2's. They are lumped together for these purposes. The administrations of the test were about ten weeks apart in the spring of 1964.

These estimates are reported in Tables IV-3 and IV-4 by separate grades for total score, component scores, and item group scores. Accompanying these estimates are the means and standard deviations on the pre-test for each score on which a test-retest reliability estimate is presented.

We had expected to find that the reliabilities would be lower and errors higher when the mean scores were near their extremes (e.g., an item group score near 0 or near 6). That is why we report the means and standard deviations in these tables.

However, a visual inspection of the reliability estimates suggests that they are relatively independent of the difficulty of the item groups. This impression is supported by the tables of standard errors of the item group scores (Tables IV-5 and IV-6). The standard errors hover around one, even though the difficulties of the item groups vary considerably.

In order to make a rough check on the relative size of the standard errors, we divided them into two groups and computed the mean standard error for each group. One group consisted of those for which the mean score on the item group was within 1.5 points of 4, that is, between 5.5 and 2.5. The other group consisted of the rest, that is those for which the mean scores were distant from our mid-cutting point. For conditional reasoning these mean standard errors were

---

* We prefer to speak of a reliability estimate, rather than a reliability, because we are securing only estimates of the actual reliabilities.

TABLE IV-3. Test-Retest Reliability Estimates on the Conditional Reasoning Test

Type of Score	Scoring Formula	Max. Poss. Score	Grade 05 N=76 Mean	Grade 05 N=76 SD	Grade 05 N=76 r	Grade 07 N=75 Mean	Grade 07 N=75 SD	Grade 07 N=75 r	Grade 09 N=64 Mean	Grade 09 N=64 SD	Grade 09 N=64 r	Grade 11 N=51 Mean	Grade 11 N=51 SD	Grade 11 N=51 r	All Grades Mean* r
Total	(R-W/2)+27	99	42.7	11.4	.76	51.2	12.4	.65	51.8	11.3	.78	53.5	14.0	.80	.75
Content															
CF	R	48	23.6	4.9	.60	27.1	5.5	.63	27.5	5.1	.62	27.8	6.7	.74	.65
SY	R	12	5.8	1.8	.48	6.6	1.9	.50	6.6	2.0	.63	7.0	1.8	.48	.53
SU	R	12	4.7	2.2	.61	5.9	1.9	.40	5.8	2.0	.62	6.0	1.9	.56	.55
Item Group															
1	R	6	4.3	1.6	.59	4.5	1.3	.56	4.8	1.3	.63	4.4	1.3	.37	.54
2	R	6	1.3	1.3	.50	1.8	1.6	.48	1.5	1.6	.61	2.3	1.7	.60	.55
3	R	6	1.1	1.2	.51	1.7	1.4	.34	1.6	1.4	.57	1.9	1.4	.45	.47
4	R	6	3.5	1.6	.54	4.0	1.5	.27	3.9	1.4	.51	3.7	1.4	.31	.41
5	R	6	3.3	1.5	.55	3.9	1.6	.58	4.0	1.6	.61	4.1	1.5	.57	.58
6	R	6	3.5	1.7	.63	3.8	1.6	.40	3.7	1.9	.50	3.4	1.5	.44	.50
7	R	6	.9	1.3	.54	1.6	1.7	.38	1.7	1.9	.69	2.3	1.8	.61	.57
8	R	6	4.1	1.6	.56	4.9	1.2	.57	5.0	1.3	.56	4.9	1.4	.64	.58
9	R	6	4.2	1.4	.50	4.6	1.2	.38	4.8	.9	.43	5.0	.9	.40	.43
10	R	6	3.4	1.7	.67	4.2	1.7	.54	4.4	1.6	.70	4.1	1.7	.44	.60
11	R	6	3.3	1.6	.62	3.9	1.4	.39	4.1	1.5	.57	3.7	1.3	.39	.50
12	R	6	1.6	1.5	.68	1.1	1.4	.55	1.0	1.1	.28	1.4	1.2	.33	.48

* Using Fisher's Z.

TABLE IV-4. Test-Retest Reliability Estimates on the Class Reasoning Test

			Grade 04 N=71			Grade 06 N=78			Grade 08 N=76			Grade 10 N=54			Grade 12 N=50			All Grades
Type of Score	Scoring Formula	Max. Poss. Score	Mean	SD	r	Mean	SD	r	Mean	SD	r	Mean	SD	r	Mean	SD	r	Mean r*
Total	(R-W/2)+27	99	43.7	15.6	.83	53.3	14.9	.88	54.8	17.9	.66	71.4	12.0	.84	70.0	16.0	.86	.83
Content																		
CF	R	48	25.5	7.6	.80	30.4	7.1	.85	30.8	8.3	.63	38.2	5.1	.79	37.9	7.4	.85	.79
SY	R	12	4.9	1.7	.44	5.5	1.7	.36	5.3	1.9	.27	6.9	1.9	.70	7.0	1.7	.62	.50
SU	R	12	4.5	2.4	.52	5.5	2.5	.70	6.2	2.8	.53	8.3	2.2	.61	7.7	2.7	.73	.63
Item Group																		
1	R	6	4.3	1.8	.60	5.3	1.1	.66	5.4	1.1	.47	5.7	0.7	.44	5.7	0.8	.77	.60
2	R	6	3.6	1.4	.43	4.2	1.3	.69	4.2	1.4	.54	5.2	0.9	.41	5.1	1.2	.65	.55
3	R	6	1.7	1.7	.64	2.5	1.8	.71	2.9	1.6	.62	4.0	1.7	.68	4.4	1.7	.76	.69
4	R	6	2.5	1.6	.45	2.6	1.4	.39	2.8	1.4	.61	3.5	1.3	.43	3.9	1.3	.68	.52
5	R	6	3.5	1.5	.48	4.2	1.1	.30	4.1	1.2	.33	4.7	0.9	.38	4.5	1.2	.41	.38
6	R	6	3.6	1.6	.53	4.4	1.4	.44	4.4	1.4	.49	5.2	1.0	.72	5.1	1.0	.38	.52
7	R	6	1.9	1.7	.51	2.3	1.8	.59	2.2	1.7	.48	3.7	1.7	.60	3.8	1.7	.46	.53
8	R	6	2.3	1.8	.57	2.6	1.7	.64	3.0	1.8	.56	4.2	1.4	.37	4.3	1.7	.45	.52
9	R	6	2.9	1.7	.50	3.4	1.5	.49	3.2	1.6	.38	3.9	1.3	.48	3.8	1.6	.48	.47
10	R	6	2.4	1.5	.32	2.6	1.4	.26	2.8	1.6	.30	3.5	1.4	.41	3.1	1.7	.50	.36
11	R	6	2.7	1.6	.44	3.4	1.5	.40	3.4	1.7	.40	4.9	1.1	.52	4.4	1.5	.51	.46
12	R	6	3.6	1.4	.32	3.7	1.3	.44	4.0	1.4	.33	4.8	1.2	.36	4.4	1.4	.58	.41

* Using Fisher's Z.

TABLE IV 5. Standard Errors of Item Group Scores - Conditional Reasoning

Item Group	N = 76 Grade 05	N = 75 Grade 07	N = 64 Grade 09	N = 51 Grade 11
1	1.02	.86	.79	1.03
2	.92	1.15	1.12	1.08
3	.84	1.14	.92	1.04
4	1.09	1.28	.98	1.16
5	1.01	1.04	1.00	.98
6	1.03	1.24	1.34	1.12
7	.88	1.34	1.05	1.12
8	1.06	.79	.86	.84
9	.99	.94	.74	.70
10	.98	1.15	.88	1.27
11	.99	1.25	.98	1.02
12	.85	.94	.93	.98

Note: If the standard error is underlined, the mean score on this item group is within 1.5 points of 4; that is, between 2.5 and 5.5

TABLE IV-6. Standard Errors of Item Group Scores - Class Reasoning

Item Group	N = 71 Grade 04	N = 78 Grade 06	N = 76 Grade 08	N = 54 Grade 10	N = 50 Grade 12
1	1.25	1.37	1.92	1.37	1.40
2	1.14	.64	.80	.52	.38
3	1.06	.72	.95	.69	.71
4	1.19	1.09	.87	.98	.74
5	1.08	.92	.98	.71	.92
6	1.10	1.05	1.00	.53	.79
7	1.19	1.15	1.23	1.08	1.25
8	1.18	1.02	1.19	1.11	1.26
9	1.20	1.07	1.26	.94	1.16
10	1.24	1.20	1.34	1.08	1.20
11	1.20	1.16	1.32	.76	1.05
12	1.15	.97	1.15	.96	.91

Note: If the standard error is underlined, the mean score on this item group is within 1.5 points; that is, between 2.5 and 5.5.

practically identical, 1.01 and 1.03 respectively. For class reasoning the mean standard errors were 1.05 and 1.32 respectively, giving a difference in the expected direction. But this is still insufficient to warrant the claim that there is an appreciable decrease in error around the crucial cutting points.

Thus we were unable to find grounds to support our expectation that the item groups would be more sensitive around the mid-range than at the extremes.

As is expected, the reliability estimates for the total scores are the largest, running from a low of .65 on the conditional reasoning test for seventh grade to a high of .88 on the class reasoning test for sixth graders. Correlations on the class reasoning test tend to run slightly higher than on the conditional reasoning test, the mean total score correlation on the conditional reasoning test being .75 and on the class reasoning test .83.

The component reliability estimates are not so high as those for the total scores, presumably because of the fewer number of items. The concrete familiar components each consist of 48 items altogether, the reliability estimates running from .60 on the conditional reasoning test for fifth graders to .85 on the class reasoning test for sixth and twelfth graders -- with a mean of .65 for conditional reasoning and .79 for class reasoning.

The symbolic components, each consisting of twelve items, had correlations running from .27 for grade eight on the class reasoning test to .70 for grade ten on the same test. The mean correlations are .53 for conditional reasoning and .50 for class reasoning.

On the suggestive component estimates of reliabilities ranged from .40 on the conditional reasoning test for seventh graders to .73 on the class reasoning test for twelfth graders. The means are .55 on conditional reasoning and .63 on class reasoning.

The item group test-retest correlations ranged from .26 for Item Group #10 of the class reasoning test at grade level six to .77 for Item Group #1 for grade twelve on the same test. The mean item group correlation for the conditional reasoning test is .52 and .50 for the class reasoning test.

Since these item group reliability estimates are based upon only six items apiece and are for scores to be used with groups rather than individuals, they are encouraging even though they average around .5. The total score reliabilities are of course more satisfactory, but have the disadvantage of representing a composite score, rather than a fairly pure score.

D. VALIDITY

Several approaches to the validity of the tests suggest themselves: Examining the items and the procedures used to develop them to see if these items do represent the subject matter that they are supposed to represent; asking outside experts to judge whether in their opinion the test does test for what it is supposed to test for; determining the test's correlations with familiar measures and seeing if these relationships make sense; examining the internal features that empirical investigation shows the test to possess; seeing how much sense one can make out of the investigations which depend upon the test; and seeing the extent to which the test correlates with some established valid measure of whatever it is that the test is supposed to measure. In this section we shall report on all but the last of these approaches to the determination of validity. The last approach, which sometimes appears as concurrent validity, and sometimes as predictive validity, we shall not use, because there is no established outside criterion for measuring what we want to measure.

1. Content Validity.

Part of our argument for content validity rests upon the procedures which we used to develop the test. We made a careful study of the field of logic, and examined different kinds of specific examples of logical reasoning in everyday life: newspaper editorials, U.S. Supreme Court opinions in support of decisions, and an auto mechanics handbook. We found no cases of class or conditional reasoning which were not covered by our principles. The principles, as was pointed out in Chapter II, are basic and comprehensive.

Furthermore we engaged in continuous consultation with members of the Cornell Philosophy Department who are interested in logic. Professor Keith Donnellan, who has the major responsibility in the Philosophy Department for knowledge of logic among graduate students, has examined and taken the tests, and judged them to be valid.

Lastly the content of the items together with the instructions is evidence of the test's content validity. Each item calls for an answer to the question which is the central question in logic: Does this statement follow necessarily? And the answers depend upon whether one has the right answer to that question in each case. Furthermore the correct answers to the items are justified by the principles. There is an obvious relation between the items and the principles, as can be seen by an inspection of the items. The relationship goes both ways. Not only do the principles imply the right answers to the items, but generally speaking, getting five or six of an item group right loosely implies a knowledge of the principles. Admittedly this last claim is not one that can be proven; it is one that we rest upon the intelligent judgment of informed interested people.

2. Construct Validity.

In looking at the construct validity of these tests, we are looking at the degree to which they make sense in their internal and external relationships. This makes our concern very broad, broad enough to include concern with content validity as well. But since that has already been discussed, we shall limit the present discussion to the other elements of construct validity.

a. Correlations with Familiar Measures. Because knowledge of logic is an intellectual trait, we expected to find a substantial correlation between logic test scores and IQ scores. Furthermore it is partly a verbal intellectual trait so we expected to find at least some relationship between logic test scores and socio-economic status (SES). Presumably knowledge of logic is something that increases with age (Burt, 1919; Piaget, 1950, 1958, 1959; Hill, 1961) - until maturity anyway - so we expected a substantial correlation with chronological age for our subjects when the various grades are grouped together. We did not expect to find any sizeable relationship between age and logic knowledge when figured at each grade level separately.

Although we expected sex to be unrelated to logical ability, we were not sure. Burt (1919), Miller (1955), and Hill (1961) found no relationship, but Sweeney (1953) found that men did significantly better on logic tests, though verbal intelligence was controlled.

We obtained correlations with these familiar variables that were basically in agreement with our expectations.

In Table IV-7, which is based on pretests for LDT's and LNDT-1's combined, one can see, substantial correlations between IQ, grade held constant, and both tests. This is also the case for chronological age when grades are lumped together. The correlations with age by separate grade levels are small and generally negative, two of the nine being significant at the .05 level. This is insufficient proof of a small negative relationship, but should there be one, it would not be surprising. In fact it would be rather interesting, for it would suggest that in a given grade the more able logicians are the younger ones. This in turn suggests the explanation that inherent mental ability is a more significant factor in logical ability than it is in grade placement. And this in turn fits in with the substantial relationships that were obtained with IQ. These speculations bear further investigation.

There are generally small positive correlations between SES and logical knowledge, six of the nine correlations being significant at the .05 level. The correlations between the tests and sex are small and on either side of zero. There does not seem to be any relationship.

TABLE IV-7. Correlations Between Total Scores on the Cornell Deduction Tests and Certain Familiar Variables: Chronological Age, I.Q., Socio-Economic Status, and Sex.

Grade	N	(Separate Grades) CA	(Grades Combined) CA	IQ	SES	SEX
			Conditional Reasoning			
05	53	-.09	.58	.61	.24	.00
07	47	-.09		.64	.28	.17
09	40	-.24		.70	.13	.17
11	48	-.05		.35	.12	.09
5,7,9, & 11	188	-.11		.58	.20	.10
			Class Reasoning			
04	47	-.36	.68	.73	.35	.00
06	58	-.07		.67	.36	.25
08	49	-.12		.25	-.10	-.05
10	40	-.26		.26	.26	-.09
12	41	.23		.50	.40	-.22
4,6,8,10, & 12	235	-.12		.52	.26	-.00

Notes:

1. These are based upon the pre-test scores of the LDT's and the INDT-l's.
2. A correlation that is significant at the .05 level is underlined.
3. For SES, signs have been reversed for ease of interpretation. A positive correlation on this chart indicates that those with high SES's did better than those with low SES's.
4. Numerical assignment for sex gave boys a one and girls a two. Hence a positive correlation suggests that girls were better in that group.
5. Details about instruments used can be found in Chapter III.
6. The correlations for CA for grades combined are based only upon those members of the LDT's who were used for computing the multiple regression equations discussed in Chapter VI. For conditional reasoning, N = 64; for class reasoning, N = 82.
7. Correlations were averaged via Fisher's Z.

b. Item Analysis. Difficulty and discrimination indices for each item at each grade level were computed. The difficulty index in this case is the percent of students taking the test who answered the item correctly. A low index signifies a difficult item.

The discrimination index shows how successfully the item discriminates, using in this case the total score on the test as a criterion. It is secured by separating out the top and bottom 27% of a group using the above criterion. The percent of subjects in the bottom 27% who correctly answer the item is subtracted from the corresponding percent in the top 27%. In general the larger the result, the more discriminating is the item, unless the difficulty indices are near the extremes.

Tables A-1 and A-4 in the Appendix list the difficulty and discrimination indices for each item at each grade level for all subjects on the pre-test and in addition, for the LDT's on the pre-test and the post-test. These indices are arranged according to item groups and the mean for each item group is presented as well, giving an idea of the comparative difficulty and discriminating power of the item groups and perhaps the principles. The discussion in Chapters V and VI of the natural cultural development of logic knowledge and the capacity for logic knowledge will draw heavily upon these tables. Here we shall make only the most obvious comments and present a few interesting means.

Tables IV-8 and IV-9 in this chapter provide something of a summary of this detailed difficulty and discrimination information, by giving the means on the pre-tests for each item group, component, and total at each grade level. This summary, when it is examined from the point of view of the development of understanding of logic, is quite revealing, but that is the topic of the next chapter.

Let it be noted, however, that there is considerable variation in the difficulty levels of the various item groups, with a range of 14.1 - 87.6 on the conditional reasoning test and 30.2 - 95.4 on the class reasoning test. These

TABLE IV-8. Mean Difficulty and Discrimination Indices For The Conditional Reasoning Test, Based Upon Pre-Test of LDT's, LNDT-1's, and LNDT-2's Combined.

	Mean Difficulty Indices				Mean Discrimination Indices			
Grade	5	7	9	11	5	7	9	11
N =	102	99	80	78	102	99	80	78
Item Group 1	71.4	74.7	77.3	78.8	37.0	32.1	18.9	34.9
2	22.7	27.4	24.8	35.3	-17.9	16.7	5.3	27.8
3	17.7	26.8	31.5	35.7	-13.0	6.2	3.8	29.4
4	55.7	68.7	59.6	65.0	50.0	33.9	24.2	25.4
5	56.7	67.9	66.3	76.1	37.0	43.2	24.2	35.7
6	56.5	66.2	57.7	65.3	50.6	30.8	13.6	14.3
7	14.1	24.1	28.6	43.2	12.3	9.3	22.7	53.2
8	68.6	80.6	77.9	85.7	47.5	35.2	31.8	30.9
9	70.8	78.3	78.6	87.6	39.5	33.3	21.9	23.0
10	54.7	71.2	68.8	77.6	36.4	46.3	39.4	46.0
11	54.6	66.0	64.2	66.2	51.9	30.2	28.8	23.0
12	26.3	17.8	19.4	21.6	6.2	6.2	3.0	5.6
CF	48.9	55.4	55.2	61.8	23.2	26.3	19.4	32.0
SY	48.1	55.8	53.4	59.5	29.0	26.5	21.2	19.1
SU	41.3	53.4	53.3	59.9	34.3	26.5	22.3	31.8
Mean over all items	47.5	55.8	54.6	61.5	28.1	27.0	19.8	29.1

Mean Discrimination Index for All Grades on Total Test: 26.0

Total N = 359

TABLE IV-9. **Mean Difficulty and Discrimination Indices For The Class Reasoning Test, Based Upon Pre-Test of LDT's, LNDT-1's, and LNDT-2's Combined.**

	Mean Difficulty Indices					Mean Discrimination Indices				
Grade	4	6	8	10	12	4	6	8	10	12
N =	94	103	100	75	72	94	103	100	75	72
Item Group										
1	76.1	90.6	91.5	93.4	95.4	53.3	22.6	21.0	5.0	10.0
2	55.7	64.9	67.7	82.4	82.7	32.7	27.4	37.0	15.0	20.0
3	30.2	42.9	49.2	66.5	75.0	24.0	40.5	39.5	39.2	45.0
4	43.4	42.6	47.7	58.0	66.7	20.7	23.8	31.5	25.8	34.2
5	64.7	70.7	66.3	80.0	79.9	30.0	14.3	19.7	10.8	22.5
6	61.7	76.9	79.0	86.5	87.1	40.0	26.8	38.3	15.8	25.8
7	33.9	38.0	36.5	58.7	63.0	6.7	44.7	37.7	31.7	46.7
8	37.1	42.4	51.7	67.6	71.6	28.0	41.7	43.2	38.3	45.8
9	51.1	56.3	58.5	66.7	58.1	41.5	20.8	32.7	17.5	26.7
10	43.6	48.2	52.5	59.3	55.3	30.0	31.6	39.5	20.8	20.8
11	52.1	62.3	66.3	79.3	78.3	38.7	27.4	45.7	19.2	38.3
12	41.7	43.9	53.2	61.1	59.7	23.3	26.8	42.6	27.5	41.7
CF	54.5	64.8	67.2	78.5	77.3	32.3	28.9	32.5	19.2	29.0
SY	43.1	44.8	46.4	55.0	61.7	22.3	20.5	25.0	24.6	28.3
SU	40.1	47.2	56.2	67.3	67.0	33.1	40.0	48.2	28.3	44.6
Mean for all items	49.3	56.6	60.0	71.6	72.7	30.7	29.0	35.7	22.2	31.5

Mean Discrimination Index for All Grades on Total Test: 29.8.

Total N = 444.

ranges extend across grades, but one can see that certain item groups are more difficult than others. This is in conformity with the view that certain principles of logic are more difficult than others and are learned later. It is in conflict with the view that one kind of logic is learned at one stage in life and another kind learned at a later stage. It appears that parts of one kind are learned before parts of the other and that parts of the other are learned before parts of the one.

The level of the mean difficulty indices (ranges: 47.5 - 61.5 for conditional reasoning and 49.3 - 72.7 for class reasoning) suggests that neither test, when administered for purposes of securing a total score, is too difficult for grades 4-12.

Our purpose in developing this test, however, was not primarily that of securing a total score, but rather to enable us to determine whether a certain principle of logic has been mastered at a given level. Hence we do not approach the test with a preconceived idea of what the mean difficulty levels of the item groups should be. Instead we in a way seek to find out what the mean difficulty levels are in order to find out what is mastered and what is not.

The discrimination index pattern, in conjunction with the difficulty index pattern, is quite interesting from the point of view of development, but as we said, that is the topic of the next chapter. The following observations are relevant here:

1. The only item group that consistently fails to discriminate at all levels of administration is conditional Item Group #12. This may be because of its great difficulty (mean always below 30%). With instruction in logic incidentally, as will be seen in Chapter VI, the item group becomes considerably easier in grade eleven with a mean difficulty of 62%.

2. Other cases of low mean discrimination indices are found at either the early grades or the advanced grades, the former apparently because the item

group is difficult at that level, and the latter because the item group is quite easy at that level. Cases of the former are conditional numbers 2, 3, and class number 7; cases of the latter are class numbers 1 and 2.

3. All low discrimination indices are thus accounted for except the fairly low ones in grades nine (13.6) and eleven (14.3) for Item Group #6 in conditional reasoning. We do not have an explanation for these fairly low indices, nor for the fact that they are so much lower than the index for this item group in grade five (50.6). However, they are not embarrassingly low; they are just fairly low.

With this one exception, then, all low discrimination indices are accounted for by the purpose of the tests, which was to ascertain whether members of a set list of basic principles were mastered at various levels. Unsurprisingly there are some which at a given level are either easy enough or difficult enough to result in low discrimination indices.

Given the purpose of these tests, the discrimination and difficulty index patterns are satisfactory.

c. Making Sense in the Context of the Study. A third type of argument for construct validity tries to show that the results of the use of a particular instrument make sense. Thus in a way the argument for validity of these instruments depends in part on the intelligibility of the experimental findings. Hence the next two chapters, which present the experimental findings in the areas of natural-cultural development and readiness development, are implicitly discussions of the construct validity of the tests. Roughly speaking the more intelligible the results, the more valid the test is shown to be.

E. SUGGESTED FURTHER DEVELOPMENT

Several ideas for further development of the Cornell Deduction Test Series (or of some other deduction test series) have occurred to us before and while appraising these two tests:

1. Tests of other kinds of deduction should be developed. Ordinal reasoning and other kinds of sentence reasoning (alternation, disjunction, conjunction, and combinations of these with each other and conditional reasoning) should be developed first, because the principles for these types are fairly well developed. Tests for deontic, alethic, epistemic and other forms of logic must await the preparation of a set of principles for each of these types. Hence the second recommendation.

2. Principles of types of logic other than class, sentence, and ordinal should be formulated. This is a very difficult task.

3. Satisfactory methods should be developed for using the two existing tests with lower elementary students (grades 1-3). One possible adjustment is to shorten the tests. Each test is designed so that the second half (Items 43-78) can be dropped off without destroying the item group structure. Instead six of the more difficult item groups are simply omitted. Tables IV-1 and IV-2 show which groups would thus be omitted.

During the pilot year of the project, we developed what seemed to be promising procedures for administering early editions of deduction tests to lower elementary students. For second and third graders items were simply read aloud while they followed along and marked answers directly on the test booklets. Testing time was broken up into separate periods of twenty or thirty minutes apiece. Two people handled classes of twenty to twenty-five students in this manner.

For first graders five students at a time were tested for periods of twenty minutes, the first of these periods being devoted simply to instructions and practice problems. A separate answer sheet was used (see Appendix) and the questions were read to the children. One person handled the testing but was very busy.

On the basis of this experience we feel that group logic testing with this sort of test is possible at the lower elementary level. Incidentally Hill (1961) tested lower elementary students in logic, but did not face the problem, which, as we indicated early in this capter, should be faced, of getting subjects to judge whether an argument is invalid or not, when the proposed conclusion does not contradict the premises.

4. Scores on these tests should be correlated with other measures of logic knowledge, such as grades in an elementary logic course and scores on other logic tests. It would be particularly interesting to see the relationship between these tests and an open-ended logic test in which subjects are asked to supply the conclusion themselves, the intent being to see the extent to which a multiple choice test like these can be used as evidence of ability to deduce conclusions, given only the premises. Thus one could see the extent to which these tests fail to test for deductive creativity (if they fail at all).

5. These tests could be expanded so as to include more items in each item group, thus achieving a more reliable measure of each principle, but of course sacrificing either comprehensiveness or compactness. Such a sacrifice will be necessary if one wants to use the test to measure and predict individuals' scores, as can be seen from the item group reliability estimates.

6. The tests could also be expanded to include a negation component. Hill found that negation was an important source of difficulty in logic for lower elementary students (1961, p. 66). In a way the conditional reasoning test has a negation component built in by means of the CF4 items. These had mean difficulties which were lower at each grade level than the total mean difficulties and the mean difficulties for each of the components. CF4 mean difficulties at grades 5, 7, 9, and 11 are respectively 38.4, 45.8, 43.3, and 50.0 as compared with total mean difficulty indices of 47.5, 55.8, and 54.6, and 61.5. Hence negation appears to be an important source of difficulty.

7. Much correlational work could be done in an attempt to see relationships between various personality and other variables and the logic components. One might wonder whether an authoritarian personality, for example, is relatively poorer on the suggestive component than is some contrasting personality type.

F. CHAPTER SUMMARY

This chapter commenced with a description of the basic structure of the tests by presenting the three components (concrete familiar, symbolic, and suggestive); showing the distribution of these components within each group of six items; showing the relationship between the item groups and the principles of logic which were presented in Chapter II; and explaining the item arrangement and format. Next operational definitions of 'mastery of X principle' were developed. Roughly speaking these operational definitions specified getting correct at least five out of an item group as a sufficient condition for mastery and getting at least four as a necessary condition for mastery.

Next came the presentation of reliability estimates, which consisted of test-retest correlations. Split-half correlations were not used, because it was not possible to split the tests into equivalent halves. These correlations were reported for total score, component scores, and item group scores at each grade level that took each of the two tests. The mean reliability estimate on "The Cornell Class Reasoning Test" is .83, and is .75 on "The Cornell Conditional Reasoning Test". The mean correlations for the components and item groups are lower, but sufficient in our judgment for use of the tests with groups.

Discussions of content and construct validity came next, with the latter consisting of three parts: correlations with well known variables, item analyses, and making sense out of the experimental results. In effect, this last topic is the problem of the next two chapters on the experimental results.

Finally a number of suggestions for further development of logic tests were made.

Chapter V. The Natural-Cultural Development of Knowledge of Logic

The development of knowledge of logic under contempory conditions is the topic of this chapter. We shall not attempt to make any estimation of the degree to which development is attributable to nature as opposed to nurture. We shall simply try to describe what we have found in this group of 803 upper New York State students whose ages range from about 9 to 18, and whose mean IQ is roughly 114. We do not doubt for a moment that environment has played a significant role in the development of their logical capacities.

One cannot, for example, be sure that there has been no previous conscious effort to teach them logic. On the contrary it is presumable that most students have received at least some informal instruction from their parents, their teachers, or their peers. But we do know that our staff did not teach logic to these students prior to the test administration which this chapter considers. In the next chapter we will consider what later happened to those students to whom we did try to teach logic.

A. THE LITERATURE

Jean Piaget is the best known and most prolific contributor to the literature on the development of knowledge of logic. First we shall examine his characterization of the formal operational stage, which he holds runs from ages 11-12 on. Then we shall pose questions which his views suggest and on which the pre-test administration of the Cornell Deduction Tests throws some light. Other studies will then be considered for their bearing on these questions.

1. Piaget's Formal Operations.

A standard claim made by Piaget is that the concrete operational period of thought runs from ages 7-8 to 11-12, at which time it is followed by the formal operational period, the apex in the development of thought (e.g., Inhelder and Piaget, 1958, p. 1*). In the approaching discussion we shall emphasize formal operational thought, but shall draw upon his notion of concrete operational thought for purposes of contrast.

a. *The Real Versus the Possible*.

According to John Flavell in *The Developmental Psychology of Jean Piaget*, "The most important general property of formal operational thought, the one from which Piaget derives all others..., concerns the *real* versus the *possible*." (1963, p. 204). Apparently it is the ability to deal with the possible, rather than just the real, that supposedly is uniquely found in the formal operational period. We have found in Piaget three, perhaps four, different basic features of this cognitive ability to deal with the possible, instead of just the real:

1) The first is the ability to judge an argument on the basis of its *validity* as opposed to judging it on the basis of one's *belief about the truth* of the conclusion. We shall call this the 'truth-validity characteristic'. It is manifested in several ways: by reasoning from assumptions which are known to be false, by reasoning from assumptions whose truth status is not known, and by reasoning and concluding without regard to the truth status of the conclusion.

According to Piaget:

> Childish reasoning beteween the years of 7-8 and 11-12 will therefore present a very definite feature...: reasoning that is connected with actual belief, or in other words that is grounded on direct observation, will be logical. But formal reasoning will not yet be possible. For formal reasoning connects assumptions -- propositions, that is, in which one does not necessarily believe, but which one admits in order to see what consequences they will lead to (1928, pp. 250-51).

---

*Henceforth in this chapter, as in Chapter II, we shall refer to this work by citing the year, 1958. According to the distribution of authorship reported in the Preface (p. *xxiv*), the parts that we are quoting and referring to are actually written by Piaget.

More specifically Piaget says the following things: The formal operational thinker is able to reason "on the basis of assumptions which have no necessary relation to reality or to the subject's beliefs... He relies on the necessary validity of an inference as opposed to agreement of the conclusions with experience" (1950, p. 148). "To reason formally is to take one's premises as simply given, without enquiring whether they are well founded or not...; belief in the conclusion will be motivated solely by the form of the deduction"* (1928, p. 251). During the concrete operational period, on the other hand, "the child cannot reason from premises without believing in them. Or even if he reasons implicitly from assumptions which he makes on his own, he cannot do so from those which are proposed to him" (1928, p. 252).

Flavell says, "In general, Piaget finds that contrary-to-fact 'what if' suppositions...tend to be foreign to the thought of middle childhood" (1963, p. 208). Flavell is here emphasizing a striking part of the truth-validity characteristic, that of working from assumptions which one actually disbelieves.

The suggestive component of the Cornell Deduction Tests is aimed at this truth-validity characteristic. In each item in this component the validity status of the argument is different from the truth status of the conclusion, and often the premises that are offered are obviously false. To identify these items in the tests, which may be found in the Appendix, see Tables IV-1 and IV-2 in Chapter IV for the item numbers assigned to this component.

2) A second feature of the ability to deal with the possible is the ability to operate in the framework of all the possible combinations in a given situation. Since this characteristic was discussed at some length in Chapter II, one quotation should suffice here:

---

* Presumably here he means 'belief that the conclusion follows necessarily'; not 'belief that the conclusion is true'. The latter is suggested by his choice of words, "belief in the conclusion".

> If we accept the task of describing the structures which actually operate in the subject's minds, we have to use the criteria furnished by the combinatorial system in distinguishing between concrete ... and formal ... operations (1958, p. 280).

Chapter II attempts to clarify Piaget's notion of a combinatorial system.

Presumably the Cornell Deduction Tests do not test for this characteristic of the ability to deal with the possible. If, however, doing conditional reasoning, which implies doing propositional reasoning, is alleged to imply working within a combinatorial system (and, as we pointed out in Chapter II, there is some suggestion to this effect), then the entire conditional reasoning test is a test of this combinatorial characteristic. We do not think that Piaget would want to allow this, so, until told otherwise, we shall maintain the presumption that the Cornell Deduction Tests do not test for this characteristic.

3) A third feature, as Piaget views the real-possible distinction, is the ability to control variables in (presumably) an empirical investigation:

> Two discoveries found at the beginning of the formal level are (1) that factors can be separated out by neutralization as well as by exclusion and (2) that a factor can be eliminated not only for the purpose of analyzing its own role but, even more important, with a view toward analyzing the variations of associated factors. (1958, p. 285)

The Cornell Deduction Tests do not test for this characteristic, unless it is alleged that this characteristic and conditional reasoning ability are logically interdependent. Because Piaget seems to relate all of the features of the formal stage to each other in a manner that is never clearly specified, there is some ground for thinking that Piaget might allege the above logical interdependence. But again, until told otherwise we shall assume that Piaget would not consider the Cornell Deduction Tests to be measures of ability to control variables.

4) Whether the fourth feature should be considered part of the real-possible distinction or a separate feature of the concrete formal distinction,

is problematic. This fourth feature is the ability to do propositional logic, instead of just class logic. According to Flavell, as quoted above, the real-possible distinction is "the one from which Piaget derives all others", so presumably the class-propositional distinction should be derived from it as well. Furthermore, as we indicated in Chapter II, there is reason to believe that Piaget wants to connect ability to do propositional reasoning with the ability to deal with the possible instead of just the real.

But, as we also indicated in Chapter II, to hold this view about propositional logic is to hold a different notion of propositional logic than the standard one and than the one which is operation in this project. The most charitable interpretation of Piaget's views is that propositional reasoning is definitionally independent of ability to deal with the possible. We shall adopt this interpretation.*

In any case Piaget maintains that propositional reasoning (and thus sentence and conditional reasoning) is not possible until age 11-12. "The child at the concrete level (stage II: from 7-8 to 11-12 years) cannot yet handle... propositional logic...." (1958, p. 1).

The conditional reasoning on "The Cornell Conditional Reasoning Test" corresponds to much but not all of Piaget's propositional logic. Omitted, because of time and space pressures, are alternation, disjunction, and conjunction. Included are conditionals, which, roughly speaking, are statements containing the word 'if' and its synonyms. That Piaget finds conditionals to be a central part of his propositional logic is seen in the following quote which, in referring to implications between propositions, is referring to conditionals:

---

*Even though we list this propositional-class distinction along with the others.

> Formal operations, therefore, consist essentially of 'implications' (in the narrow sense of the word) and 'contradictions' established between propositions which themselves express classifications, seriations, etc. (1950, p. 149).

b. Kinds of Formal Thought.

Recently Piaget has introduced two stages in formal thought which he calls IIIA and IIIB, the former running from 11-12 years to 14-15 years and the latter from 14-15 years onward (1958, p. 1). He holds that it is not until IIIB that the child thinks in terms of necessity of relations (presumably empirical relations) instead of constancy (1958, p. 11); and that the full grasp of the concept, all other things being equal, is attained (1958, p. 43). Neither of these features is a feature of formal logic and neither is tested for by the Cornell Deduction Tests.

Interestingly Piaget does not suggest, so far as we can find, that any principles of conditional reasoning are attained later than others, nor that any is more difficult than another. Similarly he does not distinguish among principles of class reasoning, nor among principles of alternation, disjunction, and conjunction. Instead he lumps all of sentence reasoning together (under the title "propositional reasoning") and all of class reasoning together, saying that the latter can be done in the concrete operational stage but that the former cannot be done until the formal stage. There seems in his work to be no differential treatment of either type of logic, in so far as development is concerned. He does of course recognize different principles of reasoning, but for study of development and mastery, he lumps them together.

c. Stages.

Piaget talks as if there are stages in the intellectual (including logical) development of youngsters. The claims about the concrete operational period running from 7-8 to 11-12, and the formal operational period running from 11-12 onward sound as if he does think in terms of stages.

Naturally the distinction between the meaning of 'stages of growth' and 'continuous growth' is a difficult one to draw, so it is hard to put the question in a precise manner. And Piaget does not help very much, perhaps because his views are changing. In Judgment and Reasoning in the Child he speaks fairly explicitly of distinct stages:

> The evolution of intelligence is therefore not...continuous, but rhythmical; it seems at times to go back upon itself, it is subject to waves, to interferences, and to 'periods of variable lengths' (1928, p. 215).

On the other hand his introduction in The Growth of Logical Thinking from Childhood to Adolescence of two subdivisions of the formal operational period, stages IIIA and IIIB, suggests on his part a belief in more continuity of development than is implied by the above quotation.

The data of the present study has bearing only on the age range 10-18, and thus should not be considered as evidence on the claim that there is a concrete operational stage. It is some evidence on the question of the existence of stages in knowledge of logic in adolescence. But whether Piaget ever actually meant to raise this question specifically is unclear, because the distinction between IIIA and IIIB is not a distinction between types of logical knowledge.

Unfortunately the question of the existence of stages is basically an unclear question because there is lacking a criterion of how much of a leveling off is necessary for there to be a stage. The question is essentially a pragmatic question, since the answer depends on one's purposes. We interpret the question, "Are there stages?" as meaning, in effect, "Is there enough of a regular leveling off for a long enough time for the leveling to be significant for one's purposes?" Thus it is quite important to specify the purposes. Out of some purposive context, the question is at least partly indeterminate. Perhaps that is why there is so much dispute about Piaget's alleged stages.

The concept, stage of development, is a tricky one. We do not feel that justice has been done it in the preceding paragraphs. Awaiting a good analysis of this concept we shall limp along with the above intuitive notion of purposiveness. Perhaps this will do, given the current lack of thorough and detailed information.

d. Questions Posed by Piaget's Work.

The age range covered by the present study (roughly 10 to 18) corresponds approximately to that of Piaget's formal operational period (11-12 onward). Because his work on this period is so extensive and well-known, we presented his theoretical description in detail in order to show the taking-off point for the natural-cultural part of our study. Of the four basic features of the formal operational period (possession of the truth-validity characteristic, ability to operate within the framework of a combinatorial system, ability to control variables, and ability to do propositional logic), our findings are relevant to the first and fourth.

These two features are the ones treated in the current study because they are the ones that are clearly related to ability to do deductive logic. The other two are either peripheral or unrelated, depending on the interpretation given them.

What we have done is not to be regarded primarily as a test of Piaget's claims, but as an extension and refinement of them. By and large we find his claims to be a bit too vague and/or indeterminate for careful testing. His concepts and concerns did, however, provide us with ideas for the questions which are listed below. Although, as the reader will see, our answers to these questions still leave much that is indeterminate, they are more definite than Piaget's claims and hence are an extension and refinement of them. Of course there is a degree of testing of his claims as well.

For the genesis of the following six questions we are intellectually in debt to Piaget. These questions were suggested by his work and strike us as theoretically interesting and practically important, from the point of view of someone making decisions about the development and placement of materials in a curriculum.

1. Is there actually a development of logical ability as children grow older?

2. Does this development (if there is any) come in stages?

3. Is conditional logic mastered by age 11-12?

4. Is class logic mastered by age 11-12?

5. Is the truth-validity characteristic achieved by age 11-12?

6. Within each type of logic, is there a development of one sort of thing before another and are some things more difficult than others at a given level? If so, what is the nature of this differential development?

Of the above questions, we feel that the last is the most interesting and the one toward which this study makes an original contribution. The first question which is about whether there is development, has an unsurprisingly affirmative answer from the present study. The second question, once you are past the problem of specifying the criteria for a stage, is though still not easy to answer, very much dependent on what one regards a stage to be. Unsurprisingly the evidence of this study points toward an answer of "Partly" for Questions 3, 4, and 5. But Question 6 is the one that brings forth specific refinements in gross statements about class logic, sentence logic and the truth-validity characteristic.

2. Other Studies.

This discussion is organized around the six questions just asked.

a. Is there a development of logical ability as children grow older?

In addition to Piaget the following people, as a result of their research, contend that there is such a development: Bonser (1910), (though his tests are hardly logic tests), Burt (1919), Winch (1921), Moore (1929) and Hill (1961). Miller (1955), on the other hand, did not find development in ability to recognize fallacies in grades 10, 11, and 12. However, many of Miller's fallacies are not fallacies of deductive logic and he worked over a span of only two years; so we are inclined to feel that the weight of evidence from the literature is in support of the initially plausible view, that there is development of logical ability as children grow older.

b. Does the development come in stages?

Out of some purposive context, this question is partly indeterminate. Hence it is difficult to report others' results on the question. We can only indicate whether what they found struck them as being regular development.

Moore (1929) believes that the development in knowledge of deductive logic that he found in children of ages 6-12 was regular (and hence not in stages). Burt (1919, p. 126) and Winch (1921, pp. 138, 209, 284) found regular improvement with grade. Hill (1961, p. 51), working with students of ages 6-8, found regular development in their knowledge of logic. Since Miller (1955) found no development in ability to recognize fallacies in grades 10-12, his results are not inconsistent with the stage hypothesis. On the whole, we feel that the reports of research do not support any stage hypothesis, particularly in view of the fact that we are unable to see support for it even in the experimental findings of Piaget himself. But because of the context dependence of the question, one must not on this basis make a definite judgment about the existence of stages.

c. Is conditional logic mastered by age 11-12?

Considerable research has been done which suggests (in conflict with Piaget's claims) that children can do at least some conditional reasoning before 11-12; Bonser (1910), Burt (1919), Winch (1921), Woodcock (1941, p. 136), Hill (1961, p. 51). But the question of whether and to what degree it has been mastered by age 11-12 has not been investigated. Now Piaget does not state that it is fully mastered by the age 11-12, but he does not indicate the extent to which he thinks it is mastered after this age is attained. The two earlier-mentioned characteristics of the distinction between stage IIIA and IIIB do not help, because they are concerned with empirical rather than logical matters.

To our knowledge no pure conditional logic tests have been administered to adolescents and analyzed for the degree and kind of knowledge shown.

d. Is class logic mastered by age 11-12?

The references cited under the previous question also support the contention that at least some class logic is mastered before 11-12. And Piaget would agree with this, though one cannot be sure whether he thinks that all of the basic principles of class logic are mastered by the end of the concrete operational period. One might think so, since class logic is a characteristic of this period, according to Piaget. However, all he says is that only class logic is used in this period (1958, p. 1). He never specifically states the extent to which he thinks it is mastered, so far as we can determine.

e. Is the truth-validity characteristic achieved by age 11-12?

So far as we can determine, Piaget is the only person who has made claims about the attainment of the truth-validity characteristic before age 11-12. A number of studies of this component of logical ability are mentioned in Chapter IV, but they are all on older people. Since these studies generally show that suggestive arguments with content are handled less ably than arguments with concrete familiar content, the minimum answer suggested by these studies to the question is that the truth-validity characteristic is not fully achieved by age 11-12. So far as we can tell, Piaget does not commit himself to a degree of attainment of the characteristic, just as he does not commit himself to a degree of attainment of propositional and class logic.

f. Within each type of logic, is there a development of one sort of thing before another and are some things more difficult than others at a given level? If so what is the nature of this differential development?

There is not much evidence on this matter. Except for his attention to the truth-validity characteristic, Piaget seems to have ignored it. Hill (1961) explicitly raised the question with respect to knowledge of different principles and found in ages 6 through 8 no pattern that she could discern (p. 57). However, her test, it should be remembered, did not test for any fallacies. In class reasoning Wilkins (1928, p. 77) found the components to be in the following order of increasing difficulty for college students: concrete familiar, suggestive, symbolic, and unfamiliar.

Burt (1919), as a result of his experience developing and administering his "Graded Reasoning Tests" held that the basic mechanisms of formal reasoning are all there by the mental age of 7, and that differences are results of complexity.

> All the elementary mechanisms essential to formal reasoning are present before the child leaves the infants' department, i.e., by the mental age of seven, if not somewhat before. Development consists primarily in an increase in the extent and variety of the subject-matter to which those mechanisms can be applied, and in an increase in the precision and elaboration with which those mechanisms can operate. The difficulty of a test depends upon its complexity, that is in the main upon four points: how many connections have to be made between one idea

> and another -- only three, as in the ordinary syllogism, or four, or more? How many of these connections has he to supply himself? How closely are these connections to be knit together -- in parallel, in series, or in a more or less intricate system? How far do they fall into the same category -- of time, space, number, etc., or differ one from another? Other points, what is the precise nature of these connections -- temporal, spatial, numerical, causal, etc. -- and of their interconnections -- hypothetical, disjunctive, etc. -- are of little importance.

Burt's four aspects of complexity mentioned in the above paragraph (number of connections, number of connections to be supplied by the subject, intricacy of total, and extent of being in the same category) are potentially workable categories of research. Of course they need to be defined and clarified, but one might classify arguments according to these categories and see if subjects at various levels can handle them.

Our difficulty with these four categories of Burt's is that they do not seem immediately useful in making decisions about curriculum and grade placement of materials. The reason for this is that we now think in logic in terms of valid and invalid arguments, class and sentence reasoning, and various rules of inference. It would be possible, but inconvenient, to think instead in terms of Burt's four aspects of complexity, suitably reformulated. However, before turning to Burt's categories we should try to work with the established ones and see if differential development exists in terms of the established categories. Hence the present study is framed in terms of the established categories of logic. It would be worthwhile in the future not only to replicate this study, but to compare the fruitfulness of the two conceptual schemes.

Summary.

Piaget's discussion of four basic features of formal operational thought (possession of the truth-validity characteristic, ability to operate within a combinatorial framework, ability to control variables, and ability to do propositional logic) provided a conceptual springboard for the framing of the above six questions about the natural-cultural development of knowledge of logic. Although there will be a testing of his claims to some extent, the current study is to a greater extent concerned with the extension and refinement of his views.

The evidence of others supports the contention that there is development of logical ability; does not support the view that this development comes in stages; suggests that the basic principles of both conditional and class logic are not mastered by age 11-12; supports the contention that the truth validity characteristic is not fully developed by age 11-12; and has little to say about the differential development of principles and components of logic. The major contribution of the current study to knowledge about the natural cultural development of logic is its exploration, conceptualization, and tentative results on the differential development of knowledge of logic.

B. THE RESULTS

Like the previous section this part of Chapter V is organized in accord with the six listed questions.

1. Is there a development of logical ability as children grow older?

That our results indicate a positive answer to this question can be seen in a number of ways. In Chapter IV Tables IV-8 and IV-9, which give the mean pre-test difficulty indices for item groups, components, and total test, show a development from the lowest grade to the highest on all item groups except #12 conditional, on all components, and on both tests as a whole. Apparently Item Group #12 conditional is just too difficult for any development to show within the range with which we worked.

Inspection of total scores on the tests for LDT's, LNDT-1's, and LNDT-2's separately also reveals this development. These scores appear in Chapter III in Table III-3. Table V-1 in this chapter presents the mean total pre-test scores for each grade level, all subjects combined. Again development with advancing age and grade can be seen. For interpretation purposes this table also presents the mean chronological age, IQ, and estimated mental age. Even though the IQ scores are on different tests, resulting from the combination of the LNDT-2's with the others, these scores help to explain the larger jumps found from the 5th to the 7th grade on the conditional reasoning test and from the 8th to the 10th grade on the class reasoning test.

Table V-1 also presents the mean pre-test scores for all subjects combined on the components and item groups. Again development is in evidence in every case except for #12 conditional. If one looks more closely at these figures than simply inspecting the figures for the youngest and oldest, the significance of IQ as well as age and grade is again suggested.

There are altogether 112 different steps from one grade to the next taking into consideration total, component, and item group scores. Of these 112 steps, 16 are down, 2 are at the same level, and 94 are up. Let us examine the 16 steps down. Only one of them occurs where there is an increase in mean IQ from one grade to the next, and that is again for Item Group #12, conditional. Of the others, 6 occur in the shift from grade 7 to 9, conditional; 4 occur in the shift from grade 9 to grade 11, conditional; and the other 5 occur in the shift from grade 10 to grade 12. In the first two of these three shifts there is an actual decrease in the mean I.Q. In the third shift the mean IQ (120) stays the same. The other 4 shifts (grades 5 to 7, 4 to 6, 6 to 8, and 8 to 10) are accompanied by increases in IQ.

However, there are still some steps that do not quite fit the explanation that uses only a combination of the chronological age, grade, and IQ factors. Perhaps there is also a leveling off in grades 9 through 12 (CA roughly 15 through 18 years; MA roughly 200 through 260 months). We shall return to this possibility when considering the stage question.

TABLE V-1. Mean Chronological Age; IQ; Estimated Mental Age; and Total, Component, and Item Group Conditional and Class Reasoning Pre-Test Scores; by Grade for All Subjects Grouped Together.

	Conditional Reasoning				Class Reasoning				
Grade	05	07	09	11	04	06	08	10	12
N =	102	99	80	78	94	103	100	75	72
Chronological Age (nos.)	129	153	184	203	117	142	166	190	214
IQ	108	117	110	109	109	112	113	120	120
Estimated Mental Age (mos.) (CA x IQ/100 before rounding)	139	179	201	220	127	159	187	228	256
Total Score*	42.4	51.7	55.3	56.6	44.3	53.4	57.8	71.2	73.4
Component**									
CF	23.3	27.1	29.0	29.5	25.6	30.2	32.1	38.2	39.0
SY	5.8	6.7	6.5	7.2	5.1	5.3	5.9	6.9	7.5
SU	4.6	6.0	6.4	6.4	4.7	5.6	6.7	8.3	8.2
Item Group**									
1	4.3	4.5	4.9	4.7	4.3	5.4	5.5	5.7	5.8
2	1.4	1.7	2.1	2.2	3.7	4.3	4.5	5.2	5.2
3	1.2	1.6	2.1	2.0	1.8	2.5	2.9	4.0	4.6
4	3.3	4.1	3.8	3.9	2.6	2.6	2.9	3.5	4.1
5	3.4	4.0	4.2	4.5	3.6	4.2	4.2	4.7	4.8
6	3.3	4.0	3.9	3.6	3.5	4.5	4.7	5.3	5.2
7	0.9	1.5	2.0	2.6	2.0	2.2	2.3	3.7	3.9
8	4.1	4.9	4.6	5.2	2.3	2.5	3.0	4.2	4.5
9	4.3	4.7	4.6	5.2	2.9	3.3	3.5	3.9	4.1
10	3.3	4.3	4.2	4.5	2.5	2.7	3.1	3.5	3.3
11	3.2	4.0	4.1	3.9	2.8	3.4	3.9	4.9	4.7
12	1.6	1.0	1.8	1.3	3.5	3.6	4.2	4.8	4.6

*Total score was calculated using scoring formula: R-W/2 + 27.

**Component and item group scores are number of right answers.

Development of logical ability with grade (and thus age) can also be seen in Tables V-2 and V-3, which present the percentages of subjects at each grade who met the sufficient condition for mastery (at least five correct) of each principle or combination and who failed to meet the necessary condition for mastery (at least four correct). A reminder: Those who failed to meet the necessary condition are those who marked correctly three or fewer of the items in an item group. Hence those subjects who marked correctly exactly four of an item group neither met the sufficient condition, nor failed to meet the necessary condition. For this reason the percentages do not add up to 100%, the difference between 100% and the sum of the percentages given for a given grade being the percentage who marked correctly exactly four items in an item group.

An inspection of these two tables again shows development in logical ability. Differences among principles, which were apparent in the difficulty index tables (IV-8 and IV-9) and in the absolute score table (V-1) are again apparent in these necessary and sufficient condition tables. These will be discussed under the topic of differential development.

In summary there is quite clearly development of logical ability in subjects like those we tested. Even though this was not a longitudinal study, it would be very difficult to deny the development hypothesis and still offer a satisfactory explanation of the data.

2. Does the development come in stages?

As indicated earlier, the answer to this question depends upon what degree and length of leveling off shall count as a stage -- and this depends in part upon the general context in which the question arises. Hence for this question there are not only problems of errors and gaps in measurement; there are also problems of interpretation of the question. We are thus in a position only to make the most tentative suggestions.

Table V-2. Percent Meeting the Sufficient Condition and the Percent Failing to Meet the Necessary Condition For Mastery of Each Principle at Each Grade Level on the Conditional Reasoning Test.

	Percent Meeting the Sufficient Condition				Percent Failing to Meet the Necessary Condition			
Grades	05	07	09	11	05	07	09	11
N =	102	99	80	78	102	99	80	78
Item Group								
1	51	56	66	62	30	26	21	22
2	3	6	5	12	92	80	90	73
3	2	3	4	3	94	92	89	85
4	30	41	35	35	54	36	41	40
5	25	45	40	58	48	38	35	22
6	34	40	35	33	51	36	45	47
7	2	5	11	19	94	84	80	68
8	46	63	70	79	31	20	13	9
9	53	63	69	81	26	17	23	5
10	26	52	53	58	54	30	34	23
11	23	40	46	40	51	37	33	36
12	4	4	1	0	86	91	93	95

Table V-3. Percent Meeting the Sufficient Condition and the Percent Failing to Meet the Necessary Condition For Mastery of Each Principle at Each Grade Level on the Class Reasoning Test.

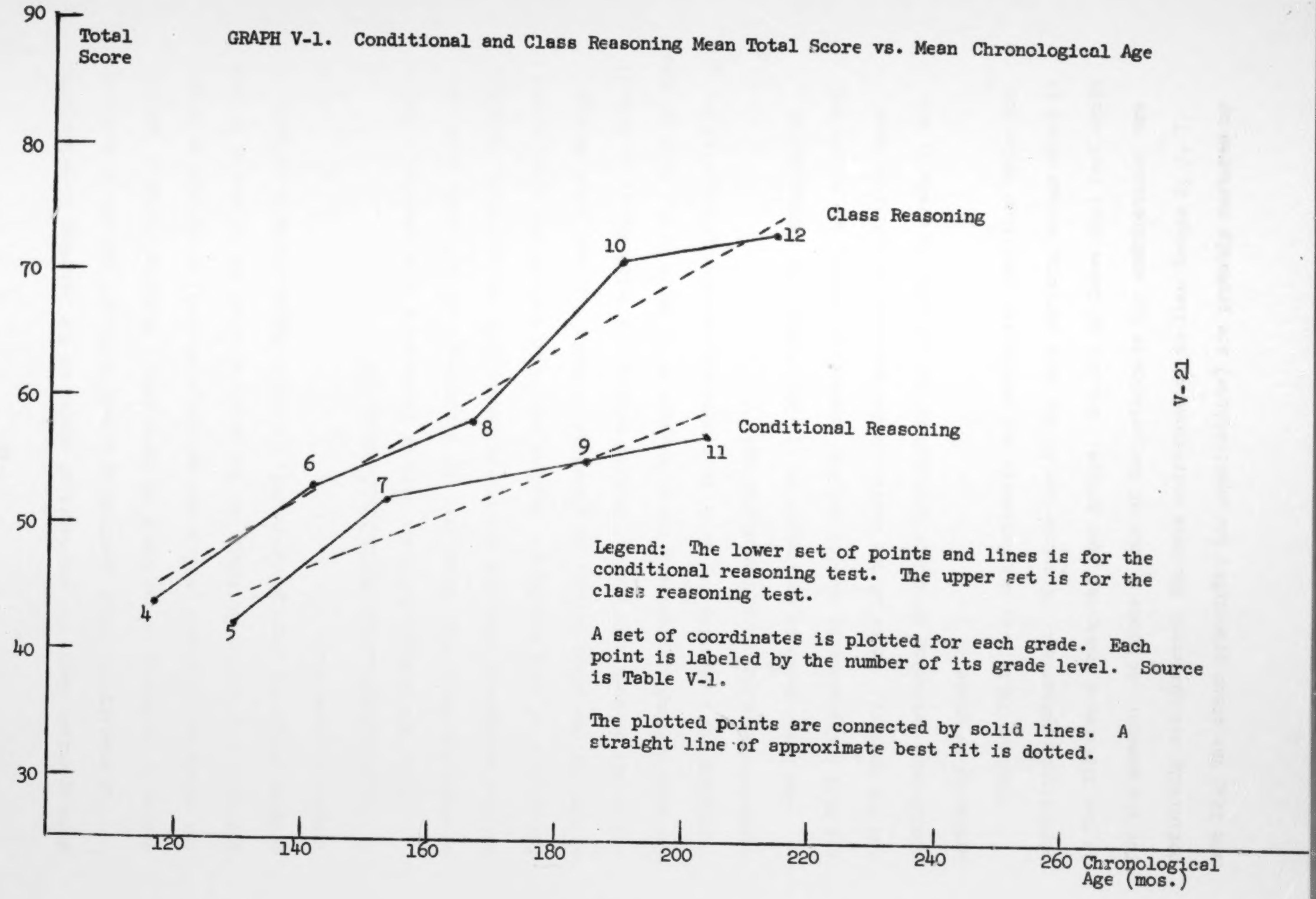
	Percent Meeting the Sufficient Condition					Percent Failing to Meet the Necessary Condition				
Grades	04	06	08	10	12	04	06	08	10	12
N =	94	103	100	75	72	94	103	100	75	72
Item Group										
1	56	79	91	92	97	33	10	5	1	3
2	34	44	55	80	75	41	28	25	4	6
3	5	15	21	44	68	83	66	63	35	22
4	10	10	15	24	43	71	72	68	48	32
5	31	37	46	63	67	40	24	24	8	17
6	30	56	63	83	85	46	22	19	5	7
7	5	15	13	35	47	83	79	73	40	42
8	11	17	24	37	58	74	73	58	36	26
9	22	20	29	39	42	63	56	52	37	29
10	10	14	22	28	25	73	74	60	48	54
11	15	29	39	64	65	67	50	38	13	19
12	30	26	47	63	58	49	45	24	12	20

Graph V-1 depicts the total score vs. chronological age coordinates for each test, making use of the means that are presented in Table V-1. The lower line connects the points for the conditional reasoning test, and the upper line those for the class reasoning test. Since the points are approximately two years apart in each case, and since there are only four and five sets of coordinates respectively, the graphs are difficult to interpret. Do they show stages or not? We find ourselves unable to give a firm answer to the question.

However, there is some indirect evidence against the existence of stages or at least in favor of their being less severe, if they exist. This is seen in a comparison of Graphs V-1 and V-2. Graph V-2 plots the same total score means against estimated mental age instead of chronological age. The points in the mental age graph come closer to fitting the speculative dotted straight line than do the points in the chronological age graph. In other words the degree of abruptness that is found in the chronological age graph can be at least partially explained by variations in IQ from one grade level to another.

Naturally this method using estimated mental age is not a precise one, since it is based upon different tests, and since the relationship between IQ score and mental age is a matter of some controversy. But the procedure does seem to have merit, so we used it. One must make some tentative assumptions.

Even with the mental age adjustment, there is still not exactly a straight line for either kind of reasoning. The data might be explained by experimental error or measurement, or the existence of stages of some sort. Conceivably there are stages for each of the principles of logic. If so, they would be masked by this total score treatment. Regrettably we do not feel that the data of this study are definitive enough to do more



GRAPH V-1. Conditional and Class Reasoning Mean Total Score vs. Mean Chronological Age

Legend: The lower set of points and lines is for the conditional reasoning test. The upper set is for the class reasoning test.

A set of coordinates is plotted for each grade. Each point is labeled by the number of its grade level. Source is Table V-1.

The plotted points are connected by solid lines. A straight line of approximate best fit is dotted.

than speculate about the existence of stages in the learning of principles.

In summary the stage question is a very difficult one and no definitive answer is warranted on the basis of these data. There is evidence against the existence of stages (within the age range studied) as extreme as Piaget posits in his earlier writing, but the question about the existence of more refined stages is largely unanswered, and until the question is refined, largely unanswerable.

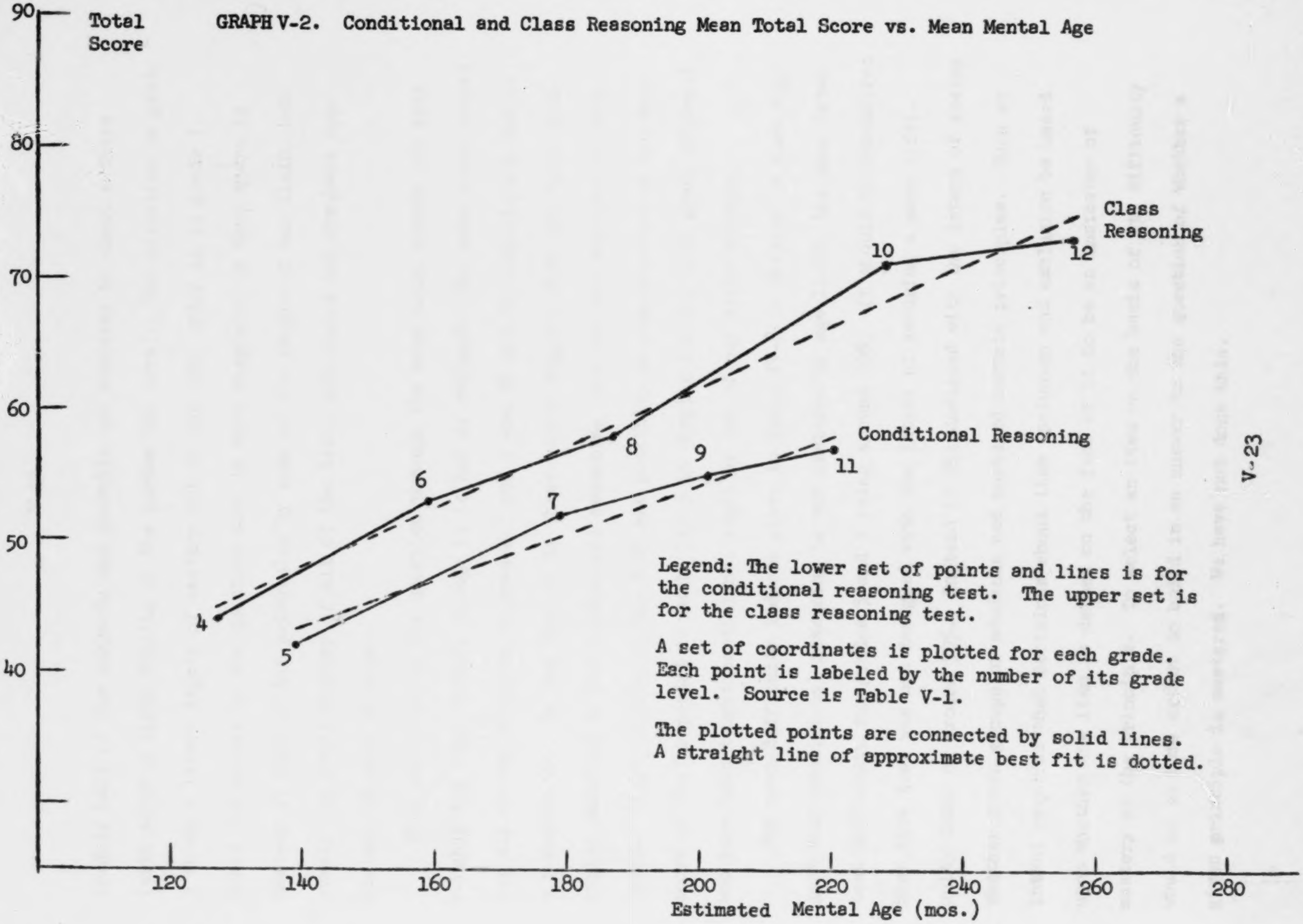
3. Is conditional logic mastered by age 11-12?

There are several different ways of approaching this question. One is by examining Table V-3, which give the percentages at each grade level who met the sufficient condition and who failed to meet the necessary condition for mastery of each principle. Another way is to look at the total scores at the various grade levels, as presented in Table V-1, and judge whether any or all of such mean scores indicate mastery. A third way is to look at the mean difficulty indices given in Chapter IV in Table IV-8. Each of these ways supports a negative answer to this question, whether interpreted in terms of chronological age or mental age.

Age 11-12 years is equivalent to 132-144 months. In chronological age this corresponds to our 5th and 6th graders, in mental age to our 4th and 5th graders. Hence if the answer to the question is positive, then conditional reasoning should be mastered by our subjects in grade 5, and certainly by grade 7.

Consider Table V-2, the necessary and sufficient condition table for conditional reasoning. Of course one of the most striking things about it is the difference among the item groups. But let us leave that fact aside for the moment. At grade 7 four of the principles (or combinations) are definitely not mastered; the ones corresponding to Item Groups 2, 3, 7, and 12. The other principles (or combinations) are probably mastered by

GRAPH V-2. Conditional and Class Reasoning Mean Total Score vs. Mean Mental Age



Legend: The lower set of points and lines is for the conditional reasoning test. The upper set is for the class reasoning test.

A set of coordinates is plotted for each grade. Each point is labeled by the number of its grade level. Source is Table V-1.

The plotted points are connected by solid lines. A straight line of approximate best fit is dotted.

roughly half of the subjects and probably not mastered by about a third (with about a sixth falling in the borderline area). The situation in grade 5 shows a lesser degree of mastery but on the same order as in grade 7. Hence the answer to the question that is here suggested is that there is mastery of some of the principles by some of the people of age 11-12, but rarely (if ever) mastery of all of the basic principles and frequent non-mastery of most of them.

When one looks at the question through the mean total scores one sees roughly the same answer, though it is not as refined. The mean total scores for all of our subjects in grades 5 and 7 are 42 and 52 respectively out of a possible 99. On the face of it these scores suggest that the basic principles embodied in the conditional reasoning test are not mastered at this level. Higher scores on the test are possible, as demonstrated by the mean score of 80.7 reported in Table III-3 in Chapter III for 11th grade students who have been taught conditional logic by one of our staff members.

The mean difficulty indices given in Table IV-8 in Chapter IV also suggest that conditional reasoning is not mastered by age 11-12, the mean total test difficulty for grades 5 and 7 being around 50%. It should be remembered that this test was not designed with the intent of securing a mean difficulty index of around 50%. Instead it was designed with the intent of seeing whether given groups of students had mastered certain principles. Such an intent requires that decisions about item inclusion and exclusion be based upon whether the item is deemed on the face of it to be an indicator of mastery of the principle. To select an item on the basis of its difficulty would be to some extent to build in an answer to the question of whether a given principle is mastered. We have not done this.

In summary, it seems clear from any and all of the three ways of presenting the data that the basic principles of conditional reasoning are not mastered by age 11-12, given the contemporary cultural background of these subjects.

Incidentally it should be noted that, although there was considerable improvement from grade 5 to grade 11, conditional reasoning is not mastered by the older group either.

The mean chronological age of our grade 11 subjects is slightly under 17 years and the mean estimated mental age is 220 months (slightly over 18 years). The same four principles and combinations are not mastered by these students. 73%, 85%, 68%, and 95% of the 11th graders failed to meet the necessary condition for mastery of the principles and combination corresponding to Item Groups 2, 3, 7, and 12 respectively. And the mean score on the total test was only 57 out of the possible 99.

This is not to say that the 11th graders are incapable of mastering the basic principles of conditional logic. It is simply that under contemporary conditions they do not do so. That they are capable of much more is shown by the performance of the 11th graders who were taught logic by a member of our staff. This is one of the striking findings to be presented in the next chapter.

4. Is class logic mastered by age 11-12?

This question will be approached from the same three vantage points. If the answer to the question is affirmative, then, considering the age equivalents mentioned under the conditional reasoning discussion, our subjects in grade 6 should have mastered class reasoning.

Consider Table V-3, the necessary and sufficient condition percentage table for class reasoning. Again differences among item groups are notable, though they are generally not so extreme as in conditional reasoning. And again some of the principles are not mastered by the majority, since over 50% fail to satisfy the necessary condition for the principles and combinations corresponding to item groups 3, 4, 7, 8, 9, and 10. Only for two principles is the sufficient condition satisfied for more than 50% of the grade 6 subjects. Thus the necessary and sufficient condition tables indicate that the basic principles of class reasoning are not yet mastered at age 11-12, whether put in terms of mental age or chronological age.

A similar conclusion can be drawn from the mean total score, which is given in Table V-1 as 53 out of a possible 99 for the 6th graders. And the mean difficulty index (58%) at the 6th grade level, which is given in Table IV-9 in Chapter IV, supports the view that the basic principles of class reasoning are not mastered by age 11-12, though of course there are some students who have mastered some of them.

Even the oldest subjects, those in grade 12 (mean chronological age of slightly under 18 years and mean estimated mental age of 256 mos., or slightly over 21 years*) have not fully mastered the basics of class reasoning, although they seem to come fairly close. For only two principles have over 80% of them attained the sufficient condition, and for five principles (or combinations) over 25% of them have failed to meet the necessary condition. Their mean score on the test is 73 out of the 99 possible, and their mean difficulty index is about 73%.

With a group at this state of advancement, the question, "Have they mastered class reasoning?" needs a more refined answer than the question explicitly requests. The answer would probably go something like this: Probably some of them have mastered all of the basic principles, some have mastered most and some have mastered only a few. The above statement is qualified with the word 'probably' because our analysis is of groups, not of individuals. The above answer would explain the data.

---

*Whether a mental age of 21 years makes sense, given contemporary IQ theory, is an interesting question, but one which we do not have to resolve here. When we are at this level, we can simply arbitrarily define 'mental age' as the product of IQ and CA divided by 100, and use the result as what we suspect is a better indicator (for our purposes) of mental development than either IQ or CA would be alone.

Thus we do not find full mastery of class reasoning even at age 17-18. Hence Piaget's characterization of the concrete operations period as one in which class logic can be done deserves qualification, if our data are taken at face value. Class logic is not, under the cultural conditions of our subjects, fully mastered during the concrete operations period. Not until some years later is mastery of the basic principles of class logic approached.

However, Piaget's contention that class logic is easier than sentence logic receives some support from the above analyses, though the stage form of his conclusion is not supported. That is, right down the line class logic scores are generally higher than conditional logic scores (this assumes that conditional logic scores are representative of scores that would be obtained on a complete sentence logic test). But it is not a matter of one being mastered at one stage, and then the other being mastered at another stage. Instead neither is fully mastered by age 17-18, and both appear to be developing fairly regularly up to that age.

Mainly because the conditional and class reasoning tests were given at different grade levels, we did not perform tests of statistical significance for the differences between performance on them, feeling that such comparisons could be made at some other time on subjects who take both tests. But Graphs V-1 and V-2 do suggest differences in overall performance on class reasoning and conditional reasoning. It is the sort of difference suggested by these graphs that supports Piaget's implicit contention that class logic is easier than sentence logic. Naturally the support must be qualified by the lack of tests of statistical significance, which we feel should wait until more directly comparable scores are available.

5. Is the truth-validity characteristic achieved by age 11-12?

The data suggest an answer of "Partly" to this question. Tables IV-8 and IV-9 of Chapter IV present the mean difficulty indices for the suggestive items, of which there were twelve in each test. These suggestive items, it will be remembered, ~~and~~ are such that the truth status of the conclusion is different from the validity status of the argument, so a person who cannot judge the validity without being swayed by his beliefs will do poorly on these items.

The mean difficulty indices for the conditional reasoning test suggestive items are 41.3 and 53.4 respectively for grades 5 and 7; hence the average suggestive item was marked correctly about 41% of the time by 5th graders and 53% by 7th graders. On the class reasoning test the corresponding figures are 40% and 47% for 4th and 6th graders. These percentages indicate that there is a degree of achievement of this characteristic, but that the achievement is not complete.

The percentages for the 11th and 12th graders on the conditional and class tests respectively are 60% and 67%. These show greater achievement of the truth-validity characteristic, but still indicate that the achievement is yet incomplete. This result is of course to be expected from the findings reviewed in Chapter IV under the topic of Wilkins' suggestive logic content.

In sum the truth-validity characteristic is partly achieved by age 11-12 and to a greater extent by age 18, though it is still incompletely achieved on the average by students of the latter age. There are incidentally interesting differences between conditional and class reasoning on this truth-validity characteristic, differences which will be discussed under the next question.

6. Within each type of logic, is there a development of one sort of thing before another. If so, what is the nature of this differential development?

Since there are virtually no suggested answers from the literature on this question, and although some of the differences that will be suggested in this section seem fairly clear-cut, this research is largely exploratory. Theoretical background and/or replication are needed.

a. Differential Development of Knowledge of Principles of Logic.

1) The Fallacies. The most striking difference among the raw scores, difficulty indices, and necessary and sufficient condition percentages for the principles is the difference between the principles which express the basic fallacies and those which express the basic validities. The principles expressing the fallacies are those which specify certain conditions which fail to entitle one to draw a conclusion of a certain type. Conditional reasoning Principles 2, 3, 7, 11, and 12 listed in Table II-1 and class reasoning Principles 3, 6, and 7 listed in Table II-2 in Chapter II are the fallacy principles. As shown by Tables IV-1 and IV-2 in Chapter IV, these principles (except for conditional 12) are embodied in Conditional Item Groups 2, 3, 7, and 12 and Class Item Groups 3, 4, 7, and 8. Their symbolic manifestation is also presented in the tables in Chapter IV. For quick identification in Tables IV-1 and IV-2 one can use the answer 'Maybe' as a sign of a fallacy item group.

All other principles are classified as 'validity principles'. They describe or indicate a logical move that one is entitled to make. There are two kinds of item groups embodying the validity principles, those in which the proposed statement follows necessarily (thus giving a valid argument), and those in which the proposed statement is the denial of a statement which follows necessarily. Items fitting the distinction between these two types of item groups are keyed 'Yes' and 'No' respectively. Hence items embodying validity principles are those keyed either 'Yes' or 'No'.

An inspection of Tables IV-8 and IV-9 in Chapter IV shows that at the lowest grade levels (4 for class and 5 for conditional) the 'Maybe' item groups have the four lowest mean difficulty indices for each of the tests. These 'Maybe' item groups are the most difficult in each test at that level. Roughly the same situation holds in the necessary and sufficient condition percentages given in Tables V-2 and V-3. Conditional Item Groups 2, 3, 7, and 12 and Class Item Groups 3, 4, 7, and 8, which are the 'Maybe' item groups, are the most difficult in each test, with one minor exception.*

At this age level (CA of 10-11; estimated MA of 11-12) students seem better able to tell that something which follows, does follow; than that something which does not follow, does not follow.

Furthermore there is generally considerable improvement among the 'Maybe' items. The actual improvement for each item group is shown in Table V-4, which is derived from Tables IV-8 and IV-9.

Table V-4 shows that the largest amount of item-group improvement for each type of reasoning is registered for a fallacy item group (conditional 7 and class 3). In class reasoning the next two largest improvements are also for fallacy groups (8 and 7), and the fourth fallacy group (4) is among the top ones in improvement. In conditional reasoning two other fallacy groups (2 and 3) are high ones in improvement; the fourth however (12) is the lowest in improvement, presumably because it is so hard. Furthermore it is not a pure fallacy group; it is the only one which embodies a combination of principles.

It appears then that although the fallacy principles are the most difficult at ages 10-12, there is great improvement in knowledge of these principles as students grow older.

---

*In the sufficient condition table for class reasoning, Item Group 10 intrudes by two percentage points.

TABLE V-4. Difference Between Lowest Grade and Highest Grade Mean Item Group Difficulty Indices; Item Group Answers

Item Group	Conditional Difference	Conditional Answer	Class Difference	Class Answer
1	7.4	Yes	17.3	No
2	12.6	Maybe	26.7	Yes
3	18.0	Maybe	44.2	Maybe
4	9.3	No	23.3	Maybe
5	19.4	Yes	15.3	Yes
6	8.8	Yes	24.8	No
7	29.1	Maybe	29.1	Maybe
8	17.1	No	34.5	Maybe
9	16.8	Yes	15.6	Yes
10	22.9	No	15.7	No
11	12.6	No	26.2	No
12	-5.3	Maybe	4.9	Yes

Note: Item group answers are given because they indicate the type of item group: 'Maybe' for fallacy item groups; 'Yes' for item groups in which the conclusion to a valid argument is offered; and 'No' for item groups in which the denial of a conclusion to a valid argument is offered.

The question inevitably arises as to whether, using these improvement figures, the actual improvement in amount of knowledge is necessarily being compared. After all there is inevitably little improvement on a test where there is little room for improvement, it might be argued.

In a way there is point to this view, but it neglects the unique feature that is built into these tests. They are intended to be tests of mastery. If someone has mastered a particular principle or skill, then there is not much room for further development of knowledge of the principle or skill, although there might well be room for other kinds of development. Given our assumption that these are tests of mastery, then the comparisons of amount of improvement are legitimate. But it should of course be remembered that there might well be improvement in other things which is neglected in the comparisons that are being made. More specifically, given our assumptions, there is on the whole more improvement in the fallacy principles than in the other principles for which we tested. This does not preclude there having been an even greater improvement in some of the principles, skills, and/or combinations thereof for which we did not test.

This defense of principle improvement comparisons does not extend to component and total score comparisons, for those are not mastery scores. In those comparisons differences in improvement scores might not reflect differences in amount of improvement in the thing being measured. Artificial test ceilings might be operative.

2) The converse. Roughly speaking a converse of a statement is the statement with the parts reversed. One basic fallacy rule is that a statement does not imply its converse. In other words:

> That all A's are B's does not imply that all B's are A's.
>
> 'If p, then q' does not imply 'If q, then p'.

These two rules are among the most basic in the practical application of logic. They are put more precisely as conditional Principle 7 and class Principle 3, which correspond to the item groups of the same number.

A striking thing is that in each type of reasoning it is the converse principle which is the most difficult at the lowest level, and in which there is the greatest improvement over the years, Conditional Item Group #7 registering a mean difficulty index difference of 29.1 and Class Item Group #3 a difference of 44.2 (see Tables IV-8, IV-9, and V-4). Similar results appear in the raw score table, Table V-1. The two elementary converse item groups start with the lowest raw scores and register the greatest absolute gain over the range that we tested.

The results are not quite so striking (though almost so) when the necessary and sufficient condition tables, Tables V-2 and V-3, are examined. In each case the converse principles tie for the position of greatest difficulty. Furthermore the simple class reasoning converse item group (#3) does register the greatest gain in percentage meeting the sufficient condition and the greatest reduction in percentage failing to meet the necessary condition. And the conditional reasoning converse item group registers the greatest reduction in percentage failing to meet the necessary condition. But it does not register the greatest increase in percentage satisfying the sufficient condition, presumably because not enough subjects even at grade 11 have mastered the converse principle.

Another interesting fact about the converse principles is that at the top grade levels in our range they had (as shown in Tables IV-8 and IV-9) high mean discrimination indices, the highest for conditional reasoning (53.2%) and among the highest for class reasoning (45.0%, which is 1.7 percentage points from the highest). Hence an understanding of the fallacy of conversion is, of the subtest factors with which we worked, among the most closely related to total performance on the logic tests.

In sum the greatest improvement within the range of levels measured in this study occurred for the converse principles of both conditional and class reasoning. These principles for ages of roughly 10-12 were the most difficult and for those around 17-18 were among the most discriminating. This is in some contrast to the contraposition principles, which we consider next.

3) The Contrapositive.

Roughly speaking the contrapositive of a statement is the statement with the parts reversed and negated. A statement does imply its contrapositive. In other words:

That all A's are B's does imply that all non-B's are non-A's.

'If p, then q' does imply 'If not q, then not p'.

These two rules are put more elegantly as Conditional Principle 6 and Class Principle 8 in Tables II-1 and II-2 in Chapter II. They are directly tested for by Conditional Item Group 6 and Class Item Group 9, as shown in Tables IV-1 and IV-2 in Chapter IV.

These item groups are of medium mean difficulty at the outset (56.5% and 51.1%) and there is a very modest amount of improvement in each case (8.8 and 15.6) compared to the other item groups in a given type of reasoning test. Each of these improvement figures is the third smallest for its type of reasoning. Furthermore at the outset each contraposition group has a high discrimination index (50.6% and 41.5%), which drops considerably by the end of the period with which we worked (14.3% and 26.7%). This is just the reverse of the trend for the conversion discrimination indices.

One wonders how to explain these differences between the conversion and contraposition principles. Perhaps the conversion principles are more the sort of thing that people can learn than the contraposition principles, ability at which is essentially native, rather than acquired, and develops early in life, if at all. That would explain the difference in the amount of improvement between the two kinds of principles. And it could explain the initially poorer performance on handling the fallacy of conversion, which might not yet be learned at the early levels. Furthermore, it could explain the greater early discriminating power of the contraposition item groups, as contrasted with the greater discriminating power later on of the conversion groups, on the assumption that learning plays a larger role in test performance as people grow older.

This explanation is in accord with our experience in teaching logic. We found that the conversion principle, when explained, was fairly easily understood by all students, but that the contraposition principle was either understood right away or not at all. Contraposition did not seem as teachable as conversion.

A second possible explanation is that these subjects over the years receive deliberate instruction in the conversion principles, and not in the contraposition principles. The fallacy of conversion is regarded as a great evil by social studies teachers. Contraposition, on the other hand, is in the experience of the writers regarded as an important tool by only a few mathematics teachers.

A third possible explanation runs as follows: The high-scoring students in the early grades answered the conversion and contraposition items on the basis of whether the conclusion feels or sounds like the premise. Given that nobody at these early ages is any good at logic, the ones who worked this way could get the high total scores and would get the conversion items wrong and the contraposition items right. In the early grades that would make the former items more difficult and the latter more discriminating. Later on students begin to reason logically and the high scorers are those who do so. They get the conversion items right because they know better, but they miss the contraposition items because a little knowledge is a dangerous thing. Hence the conversion items become easier and more discriminating, while the contraposition items become little easier and less discriminating.*

Research is needed on the reasons for this difference between the conversion and contraposition principles. It could be quite fruitful in helping our understanding of intellectual ability and development.

4) Transitivity.

A transitive relationship is one which, so to speak, passes with order preserved through an intermediary. Implication and class inclusion are transitive relationships. In other words:

---

*This explanation was suggested by Prof. Jason Millman.

> Given that p implies q, and that q implies r, we can conclude that p implies r.
>
> Given that the class of A's is included in the class of B's and that the class of B's is included in the class of C's, we can conclude that the class of A's is included in the class of C's.

The transitivity principles are #5 conditional and #2 class. They correspond to the item groups of the same number. In the class reasoning test, Item Group 11 calls for the double application of the transitivity principle.

At the outset in our range the transitivity principle item groups are of medium ease, with mean difficulty indices of 56.7%, 55.7% and 52.1%. Although they start out at this level of ease there is still considerable improvement registered for each in the range covered: 19.4, 26.7, and 26.2 percentage points respectively. On the whole their discriminating power as given in the discrimination indices is somewhat better than that of most of the item groups.

Thus the transitivity principles, although they start out fairly easy, are ones in which there is considerable improvement given the range within which these tests were given.

5) The Comparability of the Two Types of Logic.

One rather striking feature of the above discussions of fallacies, conversion, contraposition, and transitivity is that it appears that there are definite similarities between the two types of logic being studied. The thesis that suggests itself is that psychologically, although conditional reasoning is more difficult, there are basic similarities between the two types of reasoning, and that grasp of the three basic rules of conversion, contraposition, and transitivity follows similar developmental patterns in each type. This is a thesis which can only be suggested on the basis of this study. It must be checked by further investigation.

b. Differential Development of the Three Components: Concrete Familiar, Symbolic, and Suggestive.

As shown in Table V-5 there is development in all three of the components on both tests. In each case the greatest development is on the suggestive component, next the concrete familiar component, then the symbolic component.

Table V-5 also shows a very striking difference between conditional and class reasoning in the comparison of components at each grade level. In the conditional reasoning test the differences among the components are small, varied, and not statistically significant.

In contrast there are regular differences among the same components on the class reasoning test, the concrete familiar being easier. At three (6, 8, and 10) of the five grade levels, the differences between the concrete familiar and the symbolic components are statistically significant (5% level) and in the other two are nearly so. The differences between the concrete familiar and suggestive components are less marked but an inspection of the chart shows that they are regular. At grade 6 the difference is statistically significant; at grade 4 it comes within two tenths of a percentage point of being so; and at the other three grades the differences are about two-thirds of what is needed for statistical significance.

These tests of significance are the relatively conservative Tukey test.* When the less conservative t-test was performed on the above differences at each grade level, all except those at grade 4 turned out to be statistically significant. Furthermore, using the t-test, none of the conditional reasoning component differences are significant. Because of a conservative leaning, we report the Tukey test results in Table V-5, but do find the t-test results rather striking, so we mention them too.

---

*See: Ryan, Thomas, A. "Multiple Comparisons in Psychological Research". Psychological Bulletin, Vol. 56, No. 1, Jan. 1954.

TABLE V-5. Comparisons of Mean Difficulty Indices of Three Components: Concrete Familiar, Symbolic, and Suggestive.

	Conditional Reasoning				Difference	Class Reasoning					Difference
Grade	05	07	09	11	05 to 11	04	06	08	10	12	04 to 12
N =	102	99	80	78		94	103	100	75	72	
CF	48.9	55.4	55.2	61.8	12.9	54.5	64.8	67.2	78.5	77.3	22.8
SY	48.1	55.8	53.4	59.5	11.4	43.1	44.8	46.4	55.0	61.7	18.6
SU	41.3	53.4	53.3	59.9	18.6	40.1	47.2	56.2	67.3	67.0	26.9
CF-SY	0.8	-0.4	1.8	2.3		11.4	20.0*	20.8	23.5	15.6	
CF-SU	7.6	2.0	1.9	1.9		14.4	17.6	11.0	11.2	10.3	
SY-SU	6.8	2.4	-0.1	-0.4		3.0	-2.4	-9.8	-12.3	-5.3	
Diff.** Needed 3 means	22.9	27.5	26.6	26.1		14.6	18.5	19.2	19.2	17.1	
Diff.*** Needed 2 means	21.2	25.4	24.6	24.1		13.5	17.1	17.7	17.7	15.8	

*Underlined differences are significant at the 5% level.

**Tukey test of significance at 5% level. The difference given is the difference needed for significance in comparing the top and bottom of three means.

***Tukey test of significance at 5% level. If, and only if, the difference between top and bottom mean is significant is this test applicable. The difference listed is the difference needed for significance in making either of the other two comparisons.

Except at the lowest level (grade 4) the suggestive component is not so difficult as the symbolic component. The differences are not statistically significant using the Tukey test, as can be seen from an inspection of Table V-5.

One wonders why there are these regular superiorities in the class reasoning test of the concrete familiar component over the symbolic and suggestive components, but not in the conditional reasoning test. Two ideas have occurred to us, one for the CF-SU comparison and the other for the CF-SY comparison. Each will have to be checked by further research.

It might be that something analogous to the suggestive component is already a part of all of the conditional items because of their "iffy" nature. What the subject is asked to suppose is the conditional statements on the conditional reasoning test is not simply that something is the case, but rather that, on another supposition (the if-clause) something would be the case. In the class reasoning concrete familiar items, if the subject accepts the initial invitation to suppose something, then he is working with what he believes to be true -- for the purposes of the test. But with the concrete familiar conditional statements what he is working with, though he might well believe it to be true, is the implication of another supposition. Thus he is forced to think in terms of what is implied, rather than what is true.

Perhaps the above hypothesis is unsatisfactory -- either because it is false, or because it is vague. But it does appear reasonable and understandable. It would explain the difference between the two tests on the CF-SU comparison. According to this explanation, the conditional CF items are also SU items and thus have the SU difficulty built into them. Thus we would expect no difference between the CF and SU components on the conditional test. The class reasoning CF items on the other hand do not have this component built in, so we can expect a difference, on the assumption which is supported by the literature cited earlier that SU items are generally more difficult than CF items. The research that has been done has used syllogisms, which are categorized as class reasoning.

This hypothesis might also explain the apparent greater difficulty of the conditional reasoning test. One might well expect this, if something analogous to the suggestive component is built into *all* the items on the conditional reasoning test.

Turning to the CF-SY comparisons, the greater difficulty of the symbolic items on the class test is not surprising, since working with variables is presumably more difficult than working with familiar categories. On the other hand there is no comparable difference in the conditional reasoning test. This *is* surprising. One possible explanation is that the symbolic component in the conditional test does not use variables, but instead talks about the existence or non-existence of letters. That is, the items that are used are of this type:

> If there is an X, then there is a Y.
> There is not a Y.
> Therefore there is not an X.

They are not of this type, in which the letters are clearly variables.

> If p, then q.
> Not q.
> Therefore not p.

Perhaps it is the use of symbols as variables, rather than simply the use of symbols that makes for difficulty. If so, then the difference between the class and conditional reasoning tests on the comparison between the concrete familiar and symbolic components is understandable.

It appears then that the order of difficulty of these three components, if one accepts the auxilliary hypotheses mentioned above, is as follows for our subjects: concrete familiar, suggestive, and symbolic (listed in order of increasing difficulty). This order of difficulty, which appears starting roughly at ages 12-14, is the same order that Wilkins found (1928, p. 77).

C. CHAPTER SUMMARY

This chapter presents a review of the literature and the results of our study of the development of knowledge of conditional and class logic of students roughly in the chronological age range 10-18 under the environmental conditions of a contemporary upstate New York area. These students had not to our knowledge been deliberately exposed to instruction in deductive logic.

1. The Literature.

The review of the literature focused on the work of Jean Piaget. Four basic features of his formal operational period of thought, which he holds runs from age 11-12 onward, were described. They are 1) possession of the truth-validity characteristic; 2) ability to operate within a combinatorial framework; 3) ability to control variables; and 4) ability to do propositional (sentence) logic. Only the first and fourth, as we interpret these features, are basically logical.

The testing of Piaget's views, because of their vagueness, was not the primary emphasis of this chapter. Instead it was concerned with questions which are interesting, practically important, and were generated from a consideration of Piaget's interests and concepts. Hence this part of the study is to be considered an attempt at extension and refinement of Piaget's work, rather than primarily a testing of his views.

The review of the literature (including Piaget's work) suggested the following:

a. That there is a development of logical ability as children grow older.

b. That no stages in this development have definitely been identified.

c. That there is no work on the extent of mastery of conditional logic in adolescence.

d. That one might infer that Piaget thinks that class logic is mastered by age 11-12.

e. That the truth-validity characteristic is not achieved by age 11-12.

f. That there is practically no study of the different developmental patterns of different principles and components of logic.

2. Findings.

Briefly (and with many qualifications omitted) the findings are as follows:

a. In this age range there is a development of ability to do logic as students grow older.

b. If there are stages in this range, they are not noticeable at the level of refinement of our measuring techniques.

c. The basic principles of conditional reasoning are not generally mastered by age 11-12, nor by age 17.

d. The basic principles of class reasoning are not generally mastered by age 11-12, nor are they fully mastered by age 17.

e. The truth validity characteristic (the ability to consider questions of validity regardless of belief in truth of the parts of an argument) is not attained by age 11-12, nor by age 17.

f. The patterns of development and mastery of principles of logic vary, but there is considerable similarity between the two types of logic studied. The principles expressing the basic logical fallacies are the most difficult at ages 10-12, but are also the ones in which there is generally the most improvement over the range studied. The most extreme example is the principle that a statement does not imply its converse. The principle of contraposition is one which in this range starts at medium difficulty and does not become much easier for older students. The transitivity principle starts in this range at medium difficulty, but is considerably easier for older students.

g. Of the three components tested, generally the concrete familiar was the easiest; next came the suggestive; and most difficult was the symbolic.

h. Of the three components tested, the greatest difference between earlier and later ages was in the suggestive component.

i. Class reasoning appears to be easier than conditional reasoning at all levels.

3. Needed Research.

The following types of further research are called for:

a. A more refined study of the possibility of stages of development of logical reasoning ability.

b. An extension of the investigation to younger age groups, using perhaps the first half of each test.

c. This investigation should be repeated on different kinds of children.

d. Investigations similar to this one should be performed for other types of logic and for loose reasoning with these two types and the others; required first are prior investigation of these other types of logic and of loose reasoning.

e. A check on the comparative difficulty of the two types of reasoning, and the principles and components, using both tests on the same students.

f. A check should be performed on the effect of a symbol's being a variable.

g. The question of the existence of basic psychological differences between conditional and class reasoning should be investigated further.

h. Further tests should be performed on the hypothesis that mastery of the principle of contraposition is more related to inherent ability than is mastery of the principle of conversion. Perhaps controlled amounts of time could be spent teaching each at various age levels and the amounts of learning analyzed.

CHAPTER VI. The Development of Readiness to Master Logic

The topic of this chapter is to be clearly distinguished from that of the last, because we are here concerned with what students *can come to know* -- not with what they know already. First of all, because *readiness* is a key concept in this chapter, we shall present an examination of the concepts *readiness* and *readiness to master a principle*. Then, after noting the dearth of literature about readiness to master logic, we shall describe the experimental procedures, including the teaching that we did, and present the results of this inevitably limited study of the readiness question. These results will be organized around two questions:

1. What did our subjects learn?
2. Can we state what others are ready to master?

A. WHAT IS READINESS?

The forthcoming discussion of *readiness* and *readiness to master a principle* is oriented toward the ultimately practical concern, the teaching of logic. Hence some qualifications which might have to be made, were we considering these concepts in other contexts, will not be crucial here, and will not be made. For our purposes the ensuing discussion should suffice. At some other time and in some other place, a general discussion of the concept *readiness* would be in order. This is not to deny that much of what follows is general, but simply to limit the problem. This is not intended to be a definitive treatment of the concept *readiness*.

1. *Capacity and Willingness*

When one inquires whether a person is ready to do something, one is not asking whether he has done the something, and one is not simply asking whether he *will* do it, for someone might never do the thing, even though he is ready to do it. One is asking in part whether he has the *capacity* to do it. But not only this. Someone might have the capacity to do something, but be declared not ready to do it because he is not willing to do it.

To say that a person is ready to do something is to say that he has the capacity and sufficient willingness to do it, which in turn implies that he probably will do it, given suitable conditions.

2. Dispositional Terms.

In philosophy there is a technical name for terms that apply to traits which are evidenced under suitable conditions. Such terms are called 'dispositional terms'. The standard example is 'soluble'. To say that a piece of sugar is soluble is not to say that is has dissolved, nor that it will ever dissolve, but simply that under suitable conditions it will dissolve. Since readiness is a trait that is evidenced under suitable conditions, the term 'ready' is a dispositional term.

'Mastery of a principle' is also a dispositional term. If one says of a person that he has mastered a principle, then one implies that under suitable conditions, the person will behave in a certain manner. For example, if one claims that a person has mastered the principle that affirming the antecedent of an accepted conditional commits one to the affirmation of the consequent, then one is committed to the person's accepting as valid simple arguments of the form used for Conditional Item Group 1.

So far what has been said about the term 'mastery' is noncontroversial. That is, it is not controversial that a person who has mastered the principle in question should be able to do something of the sort, under suitable conditions. What might be argued is whether the person should be expected to get correct at least four of Item Group #1 on "The Cornell Conditional Reasoning Test, Form X". The particular behavioral expectations that one has might be argued, but that something of the sort is required hardly seems arguable.

3. A Double Disposition.

The combination of terms, 'ready' and 'master a principle', appearing in the phrase 'ready to master a principle', is doubly dispositional, once for the readiness concept and once for the mastery concept. Roughly speaking, to say that a person is ready to master a principle is to say that he has the disposition, given suitable conditions, to develop another disposition, which, given suitable conditions, he will show in an appropriate way.

Thus the inference path from a person's behavior to a statement about his readiness to master a principle is not a simple one. More manageable, although certainly not simple, is the inference path from a person's behavior to statements about what he has mastered. The operational definitions of 'mastery of X principle' given in Chapter IV represent a rule of thumb procedure for traveling on such an inference path, a procedure which perhaps is imperfect in detail, but we think satisfactory in general approach.

4. Elusiveness of Readiness.

In addition to this doubly dispositional feature of readiness for mastery, there is another problem in inferring from behavior to readiness for mastery. The readiness disposition is more like the disposition to explode than the disposition to dissolve. When sugar has dissolved, it has not lost its disposition to dissolve, for if we let the water evaporate, the sugar residue will dissolve again. But if some powder is explosive, then once it explodes, it will not any longer have the disposition to explode, no matter how long we wait around. Similarly, if a person is ready to master a principle -- and then masters it -- he is no longer ready to master it. Therefore we can not get direct evidence that a person is ready to master something (evidence of the sort that tells us directly that a certain powder is soluble) -- although we can get direct evidence that he was ready.

Just as we can directly find that some powder was explosive, so can we find that a person was ready. We directly find that some powder was explosive when we discover that it did explode. We discover that a person was ready to master a principle by noting that he did master it.

Thus there are some dispositions that are retained while exercised and some that are lost as soon as they are exercised. Readiness to master a principle unfortunately is one of the latter type. We are interested in knowing whether it exists before it is exercised. Once exercised, it is gone, and we are no longer interested in the fact that it was present -- except for special purposes, like this study. We want to know whether a person is ready, but we want to know this before he exercises this disposition. This characteristic of the concept readiness to master a principle obviously makes problems for this sort of readiness study.

Is there then a way to find out if a person is ready to master a principle? Ideally the way to make this discovery would be to find an identical person, provide the suitable conditions, and then see if this identical person masters the principle. If so, then the subject is ready. If not, then he is not ready.

There are obvious difficulties here: first, the identical person is unavailable; even if he were, he could not be identified; and even if he could be, it would be inconvenient to arrange to put him through the paces every time we want to see if our subject is ready. Furthermore, the phrase 'suitable conditions' is vague.

5. A Substitute for the Identical Person.

A rough compromise as a way of meeting the identical-person difficulties is the study of a range of people. Such study will call for the measurement of one or more variables which are correlated with the extent, after the suitable conditions have been provided, of mastery of the principle. These correlations could be used in the development of multiple regression equations, which would serve the function of predictor equations since the values of the variables, as determined before the introduction of the suitable conditions, would be correlated with the extent of mastery, as determined after the introduction of the suitable conditions. The worth of these predictor equations would depend on the combined strength of the relationships between the predictor variables and the variable to be predicted (extent to which the principle has been grasped).

Then one or two lines must be drawn separating that degree of achievement which is deemed to indicate non-mastery and that degree which is deemed to indicate mastery. These lines need not necessarily be sharp, but for some purposes, it is convenient to make them artificially sharp. This artificial sharpness, which is the state of the lines drawn by the operational definitions described in Chapter IV, will result in some mistakes in the treatment of borderline cases, but for practical purposes, this likelihood of mistakes must be accepted. Some of the mistakes can be avoided by deliberately introducing an area of uncertainty in the present study (exactly four is the area of uncertainty).

Thus, neglecting for the moment the difficulty inherent in the vagueness of the term 'suitable conditions', a rough scheme might be developed for determining someone's readiness to master a principle. The values of the correlated variables for a given person can be put into the prediction equation and a predicted degree of grasp of the principle comes out. On the approach (the one taken here) which makes use of an area of uncertainty, there will be two lines, which will be called the "necessary condition line" and the "sufficient condition line". If the prediction falls below the necessary condition line, then we judge that the person is probably not ready to master the principle. If the prediction falls between the two lines, then we withhold judgment. If it falls above the sufficient condition line, then we judge that the person is ready to master the principle.

On the simpler approach which does not make use of an area of uncertainty, the judgment would simply depend on whether or not the prediction falls above or below the line.

Judgments about groups would not be as subject to error as judgments about individuals. A procedure for making judgments about groups will be described later.

6. Suitable Conditions.

The phrase 'suitable conditions' in the previous analysis of readiness reminds us of the practical considerations that enter into our use of the concept. Suppose that it would take six months of full-day instruction to teach a certain level of child that affirming the consequent is a fallacy. We would be tempted to say that he is not ready to learn this fact yet, because it would be just too much trouble. On the other hand, if we could teach it to him in fifteen minutes, then we would say that he is ready. Hence if we say that a person is ready to master something, we imply that it would not require an unreasonable amount of effort to teach him.

Since people will differ on what they consider to be an unreasonable amount of effort, an ideal piece of readiness research would enable one to predict the results of varying amounts of effort and let each person judge for himself whether the effort required for a given result is reasonable.

But the difficulty of performing this task, the production of predictability for varying degrees of effort, is great. The current study has attempted to investigate the results of one given amount of effort, roughly indicated as follows: three weeks of group instruction (size about 20-30) for about 40 or 50 minutes per day, such instruction provided by a teacher with special training in logic. In a later section we will say more about the teaching that we did, but the previous statement suggests some of the dimensions of effort and to most people will suggest some that we had to ignore.

The phrase 'suitable conditions' refers not only to the amount of effort, but also to the nature of the effort. The effort must be of the right sort. As a result of this fact, a piece of readiness research like the present one is again in a difficult position. How is one to know in advance which is the right sort of effort to make? No matter what approach is tried, if it fails, there is always the possibility that another would have succeeded -- that they really were ready, if only we had tried a different approach.

The inevitable compromise is to try an approach that seems feasible to an experienced person who knows well the subject matter to be taught, or, if time and funds permit, to try several of these. But it must be remembered that, following such a course, one can much more easily declare that a certain sort of person is ready than that he is not ready. If we find that people of that sort do master the thing in question, when provided with a given sort of instruction, then we can say that a person of that sort is probably ready. On the other hand if we find that people of that sort do not master the thing in question, then it is with much less confidence that we say that such a person is not ready, because there might be another feasible and successful way of conducting the instruction. This is one of those rare cases when a positive answer is easier to give than a negative one.

In the present research we were able to provide at each grade level only one set of 'suitable conditions'. They were the best we could do, given our limits of time and funds. But the results of this readiness study apply only to the sorts of conditions that we were able to provide. This is an important fact about these results.

7. Summary.

This analysis of the concepts, readiness and readiness to master a principle, has produced a number of interesting features and some difficulties that any readiness study must face. The inference path from a subject's behavior to an assertion or denial of his readiness to master a principle of logic is a complicated one. Here is a list of the complications discussed above:

1. The concept is doubly dispositional.
2. Readiness is an elusive disposition in that once shown, it no longer exists. It is explosively dispositional.
3. Readiness is a practical notion in that the means needed to produce the mastery for which the subject is ready must be feasible.
4. Since there are possibly other ways of achieving said mastery, it is difficult to deny conclusively that a subject is ready for the mastery.

Overlooking the vagueness of 'suitable conditions', briefly the proposed analysis is as follows: To say that a subject is ready to master principle X is at least to say that he has the disposition, given suitable conditions, to develop the disposition to show, given suitable conditions, the sort of behavior that a person who has mastered principle X would show. Admittedly the foregoing analysis is circular with respect to the concept mastery, but the focus here has been on readiness. Mastery was discussed in Chapter IV and can be argued separately. Its analysis is assumed here.

Any readiness study must simplify by trying to find out just what can be acquired by certain sorts of people when they are exposed to a certain set (or limited number of sets) of conditions. After the research is done and the results announced, there is still much cautious judging to be done. The inference leap to another person is one that must be undertaken with care when the relevant factors have been examined. The judgments about the feasibility of the means selected must be made. If the subjects used did not succeed, then one must consider the question of whether better methods might yet be developed.'

Hence the results of a readiness study are likely to be rather modest.

B. THE LITERATURE

There is practically no literature on the question under consideration in this chapter. Piaget's bountiful contributions concern themselves only with what children know, not what children can learn. The only thing that we have been able to find that is relevant is the effort of Hills (1961) to see the effect on children in 1st through 3rd grades of a rather restricted way of teaching: telling children, when giving them a logic test, whether each answer is correct before they go on to the next problem. But of course Hills' main purpose was other than seeing the effects of teaching logic.

The current study is the first study of readiness to learn logic with which we are acquainted.

C. PROCEDURES

1. Assignment of Grade Levels to Class and Conditional Logic; The Pre-Test

At the beginning of the spring semester, 1964, the pre-test was administered to all subjects, who are described in Chapter III. At grades 4, 6, 8, 10, and 12, the pre-test was "The Cornell Class Reasoning Test, Form X", at grades 5, 7, 9, and 11, "The Cornell Conditional Reasoning Test, Form X". These tests are described in Chapter IV. Within a week after the administration of this pretest, formal instruction in either class or conditional logic, depending on the grade level, was commenced with the LDT's, of which there was one class (size about 20-30) at each of grade levels 4 through 12. Class logic was taught to those that took the class logic test, and conditional logic to those who took the conditional logic test.

This pre-test was given primarily in order that the scores might be used in the prediction equations. It also did serve as a control in the analysis of covariance comparisons; served to alert the LDT's to the nature of the content they were to learn; but unfortunately also presumably served to provide some logic instruction for the control group.

2. Total Teaching Effort

The initial agreement with the school system called for some member of our staff, a person trained in logic and experienced in teaching at the grade levels to which he was assigned, to take over each of the LDT classes for one period per day for 15 instructional days for the purpose of instruction in logic. Each daily period was to last from 40 to 50 minutes. Naturally minor modifications of this plan were required to fit specific situations, but it was essentially followed.

3. The Actual Teaching.

Each staff teacher was instructed to teach the logical principles roughly in order, using whatever style of teaching seemed to him to be most appropriate and going as far down the list as he could in the available time. No staff member taught more than two classes per day, and when there were two classes per day they were in the same type of logic. Thus staff teachers were provided with time to plan and to develop written exercises. Each member of our teaching staff was also a graduate student at Cornell University.

The following general procedures, which are in part based on Crombach's recommendations (1954, p 272), were followed:

a. Each principle to be taught was made explicit somewhere in the course of instruction, though it might have been near the beginning, middle, or end of the instruction aimed at that particular principle. The language in which the principle was stated, and the person (teacher or student) by whom it was stated varied from class to class and principle to principle.

b. The use of technical terminology was kept to a minimum, but was not completely avoided.

c. Frequent written exercises and many examples of varied sorts were used. Each staff member was free to develop his own exercises and examples, though they used each other's ideas. Examples of the teaching materials will be found in the Appendix.

d. A modified Euler circle system was used as a model in doing class reasoning.

e. There was no discussion of any items on the Cornell Deduction Tests, nor was practice given in the specific mode of response used in the tests.

f. Homework was given in grades 7-12.

g. Although students were provided with an evaluation of their work, the students were told that their degree of achievement in logic did not bear upon any school grades that they were to receive. They were also told that a record of their progress would be given to their teachers and principals.

h. No attempt was made to counteract the positive effects of novelty, notoriety, attention, and whatever else might go into the Hawthorne effect. There were two reasons for this: First we were trying to find out what was possible and took advantage of whatever motivation was available. Second, there were disadvantages under which our instruction suffered as a result of its not being part of the regular school program. Any compensating factors were welcome.

A rough estimate of the amount of time spent on teaching each principle at each grade level was attempted by our staff teachers. This is a very difficult thing to do, since no teacher times himself in this way and since much teaching is aimed at more than one thing at a time. Furthermore we used an Euler circle system for teaching class reasoning, making it difficult to allot the time explaining the system to any particular principle. That is, the techniques of the use of circles in doing class logic apply to all the principles; hence much of the instruction applies to all of the principles, making class reasoning time allotment somewhat meaningless, but not completely so. The principle of the symmetry of exclusion, for example, has a corresponding diagram.

We did feel obligated to give some indication of the differential effort given different principles. Educational research so frequently suffers from a lack of information about teaching that is done. We hope

TABLE VI-1 Rough ~~Teaching Time~~ Estimates of the Effort Devoted to the Teaching of Each Principle at Each Grade Level.

Conditional Reasoning

Principle	Item Group	Grade 5	Grade 7	Grade 9	Grade 11
1	1	100*	80	80	70
2	2	150	60	60	65
3	3	125	75	75	65
4	4	125	70	70	80
5	5	25	40	40	70
6	6	50	70	70	65
7	7	50	40	40	70
8	8	40	45	45	30
9	9	35	15	15	30
10	11	50	100	100	60
Combination	10 & 12	0	30	30	60

Class Reasoning

Principle	Item Group	4	6	8	10	12
1	1	140	150	90	75	85
2	2	200	225	70	75	75
3	3 & 4	160	150	160	165	165
4	5	0	75	65	70	75
5	6	40	75	60	70	70
6	7	40	50	60	50	55
7	8	20	25	65	45	40
8	9	0	0	120	120	120
Combination	10,11&12	0	0	120	120	120

*Note: Times are given in minutes. '100' stands for 100 minutes.

that the reader will take this estimated time allotment for what it is: a series of very rough and sometimes not very meaningful guesses. If we had it to do over, we would probably work entirely with principles of the circle method of doing class reasoning rather than with the more elegant, but virtually unteachable principles which we used. In any case rough estimates appear in Table VI-1. It can be seen that the greatest amount of effort at the lower grades was devoted to the lower-numbered principles, whereas for the upper grades the effort was fairly evenly distributed among all the principles.

These estimates do not add up to the total time spent in the classroom for a number of reasons. To specify a few: time was spent in organization and control of the classroom; some time was not allotted because it was used in the teaching of general notions underlying these principles; and these allotments are imprecise guesses.

4. The Post-Test.

Approximately six weeks after the conclusion of instruction, a post-test was administered to all subjects, those to whom logic was deliberately taught by members of our staff (the LDT's) as well as those to whom it was presumed not to be taught (the LNDT-1's). The post-test was the same test that was administered as a pre-test, the class reasoning test to grades 4, 6, 8, 10, and 12; and the conditional reasoning test to grades 5, 7, 9, and 11. A check was made to see if logic had been taught between test administrations to the students to whom we did not teach logic; no evidence of such instruction could be found.

D. RESULTS

The results of this readiness study are divided into two parts, one dealing with the nature and extent of learning of logic that went on in the groups to which we deliberately taught logic, and the other dealing with attempts to predict what other students could learn, given comparable instruction. Each of these parts is concerned with a readiness question. The first part asks what the students in this study were ready to learn before we taught them logic; the second asks what some other students are ready to learn.

1. General Qualifications.

Four important qualifications should be kept in mind as one reads this section. First we were necessarily working with small numbers of subjects (about 20-30 at each of nine grade levels). The reason for this was that it was important to work intensively, rather than extensively in this study. In particular it was necessary to make sure that proper logic was taught rather than something else. And it was desirable to give the staff teachers adequate time to plan, prepare written exercises, and read students' papers; they were pioneering the teaching of this subject matter at grade levels lower than those at which it is ordinarily taught.

As a result of this small number of subjects results are more erratic than they presumably otherwise would be. And also in our comparisons between the LDT's and the LNDT-1's some actual differences have inevitably failed to reach statistical significance.

In order to state the other three qualification we must first distinguish among various possible causes of improvement in scores on a test of knowledge of logic. Although these distinctions and the ensuing discussion are oversimplified as a result of neglect of interaction between factors, the points that result from the discussion do not suffer therefrom. Hence we pursue the simplified line.

Possible causes of improvement in scores are:

1) Deliberate teaching of logic.

2) The taking of logic tests which are intended to measure a student's knowledge of logic. There are two features to be distinguished here:

   a) that which results in the learning of logic.
   b) that which results in the learning of how to take this and/or other tests (test-wiseness).

3) Other school influences.

4) Influences outside of school.

5) Maturation that does not depend on contributions from the environment.

Some of these factors can be introduced at the discretion of the school authorities and some are more or less beyond their control. Factors 1 and 2 can be introduced by the school authorities. Factors 3, 4, and 5 are increasingly out of their control.

As a practical matter, when we ask whether a student is ready to learn something, we are concerned with the probable results of the deliberate introduction of factors under our control. Hence we are in this study primarily concerned with the effects of factors 1 and 2.

Since the test-wiseness part of the second factor will give spurious results, we would like to discount these. When we simply ask what our students learned as a result of teaching and look only at the before and after test results, the results of this test-wiseness factor creep in. When we ask, however, whether they have learned more than they otherwise would have, and use a comparison with a control group to help answer the question, the effect of this factor is presumably controlled for to some extent at least.

With the other part of the second factor, the learning of logic as a result of taking the test, the situation is somewhat the opposite. We welcome the operation of this factor when we ask what the students have learned as a result of teaching (although its impact is undetected to the extent that it increases the pre-test score). When we use control groups, this factor is controlled for, even though we would rather that this not happen. After all, our interest is in the learning that occurs as a result of what we do. So the results of the comparisons with the control groups mask the results of actually taking the tests, which can be standard parts of instructional procedures.

Another source of error is the operation of factors 4 and 5 on entire classes, both LDT's and LNDT-1's. Extraneous occurrences, like a loud noise while the test was being taken, the presence of a smart-aleck or two in a class, a superior regular teacher who predisposed his class to attentiveness and eagerness, the beginning-of-the-year assignment of members to a class on the basis of a criterion which is motivational or related to motivational factors, etc., are examples. That such things probably occurred and that we had no check on them other than alertness and care in avoiding them is a weakness of the study. As we will explain later, we suspect that some factor of this sort operated to the detriment of the LDT's in the 9th grade.

To have provided a statistical check on this sort of thing it would have been desirable to use a number of groups with random assignment as LDT's and LNDT-1's. Such a procedure would have been considerably more expensive than the one we followed.

In summary the general qualifications we have specified are as follows:

a.) There were a small number of subjects at each grade level.

b.) Test-wiseness will result in spurious changes when the LDT's alone are being considered, but will be controlled for in comparisons with the LNDT-1's.

c.) Learning of logic attributable to the taking of the tests will justifiably have an effect on the LDT pre- and post-test comparisons (though some of this effect on change in knowledge will be hidden), but unfortunately will be blotted out in the comparisons with the LNDT-1's.

d.) Factors affecting total class groups can without detection (except for internal consistency evidence) give spuriously high or low scores.

2. What Did Our Subjects Learn?

The answer to this question gives an indication of what they were ready to learn. Generally speaking, they did not during the instructional period learn much conditional reasoning until the upper secondary level, at which time they made a vast improvement, primarily in avoiding the fallacies. On the other hand in class reasoning moderate improvement was registered at all levels with which we worked, except the lowest. These general statements need amplification and qualification.

a. Conditional Reasoning.

1) Total Scores.

The situation in conditional reasoning can be seen in various ways, each of which emphasizes different aspects of the overall situation. First let us look at total scores. One can visually compare the mean LDT pre- and post-test total scores on the conditional reasoning test as given in Table III-3 in Chapter III. One cannot see much overall improvement until Grade 11 where the difference is 17.4 points. This is a rough unsophisticated comparison, but it has its merits.

One can also compare LDT's with the LNDT-1's on the post test, taking into account differences in the pre-test and IQ. The results of such a comparison are summarized in Table VI-2. The figures are presented in detail in Table A-5 in the Appendix. These comparisons show that in grades 5 and 7 there is no statistically significant superiority one way or the other. In grade 9 there is a statistically significant superiority favoring the students who were not taught logic, and in grade 11 there was a greater superiority favoring the students who were taught logic. The superiority of the taught 11th graders is quite striking, with a difference of 15.2 points in adjusted means on the post-test. (The superiority is 22.2 points before the means are adjusted.)

The situation in grade 9 is puzzling. Not only did the control group have a significantly higher adjusted mean, but the mean score (unadjusted) of the taught 9th graders was actually lower (2.6 points) on the post-test than it was on the pre-test. Possible explanations that have occurred to us are the following: First, it might be that at this level teaching is just beginning to take hold, but just enough to be confusing. That is, it might be that at lower levels, people work on the basis of "feel" only and that at this level the teaching that has occurred has been effective enough to interfere with the "feel" method of judging arguments, but not effective enough to provide a reliable replacement.

A second possible explanation is that the particular specimen of teaching was simply confusing. A third possibility is that the problem lies with the students themselves. It might be that there was an important difference between the two groups which did not show up on any of the measures that we used. We do know that the 9th graders whom we taught were alleged to be of high enough ability to be taking algebra, but for some reason or other were not doing so. Our staff teacher reported that they did not care whether they learned logic. It might have been a problem class. We tend to favor this the third explanation, which, if the proper one, makes suspect the findings about conditional reasoning at the 9th grade level. It was only at this level that such motivational problems were reported by members of our staff.

2) Component Scores.

As can be seen in Table VI-2 the situation is roughly the same as viewed through the component scores. The 11th grade group that was taught conditional reasoning did significantly better on all three of the components

TABLE VI-2. Statistically Significant Post-Test Differences Between Students Who Were Taught Logic and Students Who Were Not Taught Logic, Using Analysis of Covariance with Pre-Test and I.Q. as Covariates.

	Conditional Reasoning				Class Reasoning				
Grade	5	7	9	11	4	6	8	10	12
N LDT	27	24	17	26	25	25	27	22	23
LNDT-1	26	25	23	22	22	34	24	18	17
Total Score	-	-	W	R	-	R	-	-	R
Component									
CF	-	-	W	R	-	R	-	-	-
SY	-	-	-	R	-	R	R	-	R
SU	-	-	-	R	-	-	-	-	-
Item Group									
1	-	-	-	-	-	-	-	-	-
2	-	-	-	R	-	-	-	-	-
3	R	-	-	R	-	-	-	-	-
4	-	-	-	R	-	R	-	-	-
5	-	-	W	-	-	-	R	-	R
6	-	-	W	R	-	-	R	-	-
7	-	-	-	R	-	-	-	-	-
8	-	-	-	-	-	-	R	-	-
9	-	-	-	-	-	-	-	-	-
10	-	-	R	-	-	-	-	-	-
11	-	-	W	-	-	-	-	-	-
12	-	-	-	R	-	R	R	-	R

Note: The symbol 'R' is used to indicate a statistically significant difference favoring the group that was taught logic. The symbol 'W' is used to indicate a statistically significant difference favoring the group that was not taught logic. A dash is used to indicate lack of statistical significance. The 5% level was used throughout.

A word of caution: The sixteen comparisons described in any one column above are statistically dependent.

built into the test. This did not happen at any of the other grade levels, and at the 9th grade level on the concrete familiar component there is another reversal.

3) Item Group Scores.

The great superiority of the 11th graders who were taught over those who were not taught seems to lie mainly in the fallacies, but also in contraposition, basic and practical, and perhaps with affirming-the-antecedent items. By 'basic contraposition' we mean the valid move from 'If p, then q' to 'If not q' then not p'. By 'practical contraposition' we mean the valid move from 'If p, then q' and the denial of 'q' to the denial of 'p'. The former appears as Principle 6 and Item Group 6. The latter, which we have also called 'denying the consequent' (Chapter II), appears as Principle 4 and Item Group 4. The two forms are logically similar, as can be seen by saying them over to oneself.

The four fallacy item groups (2,3,7, and 12) and the two contraposition item groups (4 and 6) stand out uniquely on the covariance comparison. On all of these item groups, but only these item groups, are the 11th graders that we taught significantly better than the ones we did not teach. This finding is perhaps the most striking one of this study.

It also comes through in Tables A-7 and A-8 in the Appendix, which give item group pre- and post-test measures on the groups to which we taught logic. A portion of those tables is presented in Table VI-3, which shows the difference between pre- and post-test measures for each item group in the 11th grade. The measures are mean difficulty indices, percentages meeting the sufficient condition, and percentages failing to meet the necessary condition. As can readily be seen, these six item groups stand out, and the fallacy item groups stand out most srikingly.

TABLE VI-3. Conditional Reasoning Pre- and Post Differences for 11th Graders to Whom Logic Was Deliberately Taught.

N=26

Item Group	Improvement in Mean Difficulty Indices	Improvement in Percentage Meeting the Sufficient Condition	Increase in Percentage Failing to Meet the Necessary Condition*
1	9	15	- 8
2**	50	65	-61
3**	53	73	-69
4***	11	27	- 8
5	4	2	- 8
6***	19	39	-19
7**	27	42	-35
8	- 8	-11	15
9	-13	-15	8
10	3	8	4
11	- 1	0	4
12**	44	50	-65

*A negative number here indicates a reduction in those failing to meet the necessary condition and thus shows an improvement.

**The fallacy item groups are marked by a double asterisk.

***The contraposition item groups are marked by a triple asterisk.

Note: This table is taken from Tables A-7 and A-8 in the Appendix, which present the pre- and post-test scores and differences for all grade levels to which conditional reasoning was taught.

On the other hand there is among the students taught logic a slight worsening on the 'only-if' item groups, numbered 8 and 9. Perhaps the word 'only' was previously quite clear, but that what they were taught about the word 'if' by itself confused their understanding of 'only if'. A reference to Table VI-1 shows that about an hour was spent teaching the 'only-if' principles. Perhaps this amount of time was insufficient.

It is interesting to note that there was no improvement on the 'if-and-only-if' item group (numbered 11), even though about an hour was devoted to this concept. Perhaps again the distinction between 'if' and 'only if' was confused.

There is slight improvement on the transitivity item groups, numbered 5 and 10, but one does not know whether this represents a real improvement or not. There is a greater improvement on the most basic move of all, the affirmation of the antecedent (represented by Item Group 1), but not enough to reach statistical significance in the comparison with the students not taught logic. At all grade levels, however, there is a lack of a really striking teaching-caused improvement on this, the most basic conditional reasoning principle. This is shown in Table VI-4, and is also suggested by the lack of statistical significance of the post-test differences between the LDT's and the LNDT-1's, which lack can be noted in Table VI-2.

TABLE VI-4. Conditional Reasoning Pre- and Post-Test Differences for Principle #1, the Affirmation-of-the-Antecedent Principle.

Grade	N	Improvement in Mean Difficulty Indices	Improvement in Percentage Meeting the Sufficient Condition	Increase in Percentage Failing to Meet the Necessary Condition*
5	27	9	15	18
7	24	7	5	- 4
9	17	4	0	-12
11	26	9	15	- 8

* A negative number here indicates a reduction in those failing to meet the necessary condition and thus shows an improvement.

4) Summary and Possible Import for Teaching.

In summary we appear to have been able to teach upper secondary students to avoid the basic fallacies and to some extent to recognize basic and practical contraposition as valid moves in conditional reasoning. In so doing, however, we might have interferred with their grasp of the basic valid moves from 'only if' sentences. We seem to have had little effect on their grasp of the transitivity of the if-than relationship. At this level, as well as at earlier levels, the teaching might have had some positive effect on knowledge of the most basic conditional reasoning move, affirmation of the antecedent, but, if so, the effect was small and not statistically significant. Since affirmation of the antecedent is in a way a transitivity principle (the affirmation carries through from the antecedent to the consequent), these results fit together.

The most striking fact about the results of this teaching was its great effectiveness on total score at the upper secondary level, as contrasted with its either negative effect or lack of effect at lower levels. At some future time, the location and sharpness of the implied dividing line should be investigated.

The sufficient condition percentages suggest rather strongly that the basic principles of conditional reasoning can be pretty well mastered by the upper secondary level -- at least for students like those in this study, but that there is not much point in trying to teach conditional logic in elementary and lower secondary.* Furthermore these results suggest that the things that can be taught are the fallacies, contraposition to some extent, and perhaps the validity of affirmation of the antecedent, which might partly teachable, but also seems to develop on its own without deliberate teaching.

---

* Incidentally this sentence and others that refer to upper secondary, etc., are deliberately somewhat vague because of the difficulty of placing any kind of line on the basis of these results alone. In particular we want to avoid placing the 9th graders on either side of a line,because of doubts about the motivation of the 9th grade LDT's.

That transitivity and the valid moves with 'only if' seemed to develop on their own and that deliberate teaching did not help at the levels with which we worked is suggested by the lack of improvement in percentages meeting the sufficient condition during the teaching period at all grade levels , together with the percentages that have mastered them in the 11th grade on the pre-test: 81, 81, 88, and 96 respectively on Item Groups 5, 10, 8, and 9. These figures can be found in Table A-8 in the Appendix.*

5) Learning Words vs. Learning Logic.

An intriguing question is that of whether the great improvement registered in the four fallacy item groups represented greater understanding of logic, or whether it represented a grasp of the meaning of the word 'if' as it is properly used. It might be argued that the use of the word 'if' is commonly taken to imply what is properly meant by 'if and only if', or in other words that 'if' is taken to introduce a necessary as well as sufficient condition. The suggestion then is that we did not improve their knowledge of logic; we only changed their vocabulary.

---

* A possible objection to this claim about the teachability of transitivity at the lower grade levels is that the 5th grade staff teacher estimated that he spent no time on combinations of principles. Item Group 10 calls for either the double application of the affirmation of the antecedent principle, or for the combination of the transitivity principle and the affirmation of the antecedent principle. Since our instructor reported spending no time on combinations, how can we suggest that this particular combination is not teachable at the lower grade levels, one might ask.

In reply we would say that an effort was made to teach the affirmation of the antecedent principle and the transitivity principle, but it had no noticeable effect. Hence we conclude that the combination of one with another or with itself would not have been effectively taught at the lower grade levels.

This is an extremely puzzling issue. As Benjamin Lee Whorf has pointed out, our language, our conceptual structure, and our power to think are intimately related. But it does make sense to suggest that the distinction that we represent by the words 'if' and 'if and only if' was already known to our subjects (even though they do not use these words to represent it) and that they used 'if' to mean what we mean by 'if and only if'. Thus even if there is an intimate relationship, there is also a meaningful question.

If this verbal interpretation of the learning that went on is accepted, then the learning has less significance than otherwise supposed. Under this interpretation we were not teaching the students to think; we were teaching them a common vocabulary. Now this is important too -- in order to facilitate communication in the making of crucial distinctions -- but it is not as important as teaching them to think more clearly.

This verbal interpretation is consistent with what happened with transitivity, which seems to be not simply a verbal matter, but also a conceptual one. The hypothesis that we were teaching them the meanings of words allows that there would not be much learning of transitivity. But the learning of contraposition and what there was of affirming the antecedent is contrary to the verbal hypothesis, because these things seem to be clearly conceptual, and not simply verbal. The counter-effect of this latter evidence might be explained away be the suggestion that for these things they learned a few rules by rote and did not understand them.

As one can readily see, the question bears further investigation.

b. Class Reasoning.

The situation in class reasoning is rather different from that in conditional reasoning. The dividing line, if there is one, comes in the upper elementary levels, somewhere around age 12; but that there is a line is not clear. The teaching of class reasoning did not have much effect on the 4th graders, but generally had a positive effect on those from 6th grade up to 12th grade. This effect is not nearly as striking as the effect of teaching conditional reasoning to the 11th graders.

1) Total Scores.

The change in mean total scores for the LDT's over the interval between pre-test and post-test is positive in grades 6, 8, 10, and 12, but not in grade 4. This can be seen in Table III-3 in Chapter III. The improvements registered are 9.5, 3.8, 8.3, and 6.8 respectively. These are considerably smaller than the improvement of 17.0 points registered by the 11th graders on conditional reasoning.

In a post-test comparison with the LNDT-1's, holding IQ and pre-test constant, the LDT's in class reasoning were significantly superior in grades 6 and 12, but not in the other grades, given the degree of refinement of our experimental procedures. This is shown in Table VI-2.

2) Component Scores.

The situation is generally the same when scores are broken down into their components. As indicated on Table VI-2, there is some statistically significant superiority of the LDT's at grades 6 and 12, and also at grade 8. This happens most frequently in the symbolic component. Since in the teaching frequent use was made of such phraseology as 'All A's are B's',* this difference in the symbolic component is understandable. It is perhaps noteworthy that only in the 6th grade were the LDT's statistically significantly superior on two of the components. In no grade was this the case for three components. In grades 8 and 12, it was for one component only, the symbolic component.

---

* The letters at the end of the alphabet (e.g., 'X' and 'Y') were used in the test, not the ones at the beginning, which were used in teaching.

3) Item Group Scores.

In the class reasoning item groups there is nothing that compares with the remarkable improvement made by the 11th graders on the conditional reasoning fallacy item groups. The moderate general overall improvement from grade 6 onward, when instruction is given, can be seen in Tables A-9 and A-10 in the Appendix. As can be seen in Table VI-2, for one item group, #12, there was a statistically significant superiority, IQ and pre-test item-group score held constant, at three grade levels on the post-test. This item group embodies the most complicated logical structure on the test. Perhaps the instruction was fairly successful in providing an orderly way of dealing with complex arguments.

Exceptions to this general overall improvement during the period of teaching are Item Groups 1, 10, and 11. Let us discuss them.

Item Group 1 is a special kind of item group in that it is so basic: it tests for the meaning of 'all' and negation. It calls upon someone to judge that a statement of the form, 'At least some A's are not B's', is inconcistent with the corresponding statement of the form, 'All A's are B's.' To some extent this item group is a test of whether the subject understands what is requested of him on the test, because presumably people at these levels are fairly well acquainted with the meaning of 'all' and of simple negation. So we are not surprised by the lack of improvement under instruction on this item group.

The lack of improvement under instruction for Item Groups 10 and 11 is somewhat puzzling. Item Group 10 calls for the combination of the transitivity and contraposition principles. Simple transitivity is tested for in Item Group 2 and basic contraposition is tested for an Item Group 9. There was some improvement in each of these, and according to hypothesis above, there was improvement in ability to deal with combinations. So there is something of a puzzle. However, the pre- and post-test differences on Item Groups 2 and 9, the basic transitivity and contraposition item groups, are not large enough to make the problem serious. When the study is replicated with more subjects, one can see if the problem arises again.

It is similar with the lack of improvement in Item Group 11, which calls for the double application of the transitivity principle. The single application of the transitivity principle is tested for by Item Group 2, and there is general improvement after teaching on that item group. But again the improvements are fairly small, so it is not fully clear that there is a puzzle.

4) Summary.

There is nothing striking about the effects of teaching class reasoning. There appears to be moderate and fairly general improvement in groups from age 12 onward. Two things that stand out slightly are the improvement in handling complexity represented by Item Group 12 and improvement on the symbolic component.

c. Possible Explanations of the Differences Between Conditional and Class Reasoning.

Inevitably one wants to try to explain the differences that we found between the score patterns on the conditional reasoning test and those on the class reasoning test. Why the striking improvement in the 11th grade on the conditional reasoning test and the only moderate improvement from grade 6 upward on the class reasoning test? Why was there such a contrast between the improvements on the fallacy item groups? We can only speculate at this time.

A possible explanation of the general situation is that since class reasoning is easier, the students on the class reasoning tests had less possibility of improvement. This explanation has the weakness of not explaining why in cases where there does seem to have been room for considerable improvement, there was still not an improvement comparable to that registered in conditional reasoning. Many such cases can be found in Tables A-9 and A-10 in the Appendix. For example in Table A-9 one can see that for Item Group 3, which is a conversion fallacy group, although the mean difficulty indices on the pre-test were 34, 45, 51, and 71 in grades 4, 6, 8, and 10 respectively, the improvements were only 4, 11, 21, and 13.

A second possible explanation is that in the conditional-reasoning fallacy item groups, a great share of the learning was of the sufficient-condition meaning of the word 'if', as opposed to the necessary-and-sufficient-condition interpretation. Since there was no corresponding verbal learning for the class reasoning test, the improvement was less, the explanation holds, although the amount of actual improvement in knowledge of logic was about the same.

A difficulty with this explanation is that it does not tell why this verbal learning did not occur before the 11th grade. It might be urged that if the learning is of a new meaning for a word, one would expect 7th and even 5th graders to be able to do it. This challenge to the explanation of course has no evidence to offer that a new meaning for this word, 'if', could be grasped at such an early level. So the question is unresolved, and more research should be done.

3. Can We State What Others Are Ready to Master?

The attempt to answer this question is based on the development of multiple regression equations into which one can introduce values of certain variables and produce a predicted score. Two sets of these equations were prepared: one made use only of information which is generally available in school records; the other in addition made use of pre-test scores.

In the preparation of these equations, all LDT's who took a given logic test were grouped together. Hence grades 5, 7, 9, and 11 were combined, as were grades 4, 6, 8, 10, and 12. Equations were prepared for total score, the three component scores, and the twelve item-group scores. Since there are two tests, and since there was a set of values prepared that made use of pre-test scores as well as one which did not, there are 64 equations altogether ((1+3+12) X 2 X 2). A cross-validation estimate was computed for each equation. The non-variable values that go into these equations appear in the Appendix, Tables A-11, A-12, A-13, and A-14.

The weights given in those tables are the factors by which one must multiply the scores that are introduced into the equation. These products together with the given constant are summed to give the predicted score. Here is an example of an attempt to predict a score on Item Group 4 on the conditional reasoning test:

a. An Example of the Use of a Multiple Regression Equation for Making a Prediction.

Assume the following values of the variables, which values are the mean values for the 11th grade LDT's:

Grade: 11
Chronological Age: 199.8 months
IQ: 116.0
Socio-Economic Index: 3.5
Sex: 1.5 (male=1 female=2)
Total Score on pre-test: 63.7
Pre-test score on this item group (#4 here): 4.26

Combining these figures with the weights and constant given in Table A-11, one has the following equation:

$$\text{Predicted score} = (-.001)(11) + (.0074)(199.8) + (.0064)(116.0) + (-.127)(3.5) + (.423)(1.5) + (.0384)(63.7) + (.314)(4.26) - 1.26 = -.01 + 1.48 + .74 - .45 + .63 + 2.44 + 1.34 - 1.26 = \underline{4.91, \text{ predicted score}}.$$

This figure, 4.91, can be rounded or not, depending on what one is going to do with it.

b. Making Use of the Generated Predictions.

Naturally the utility of such predictions depends on the multiple correlation coefficients and the cross validation. These figures, together with the standard errors are given in Table VI-5. Given our primary interest in predicting mastery of the principles of logic, the corresponding multiple correlations and estimated cross validations are not high enough to warrant predictions of individual item group scores. But predictions for groups seem possible. Inserting the mean scores for a group enables one to generate a predicted mean score for the group.

But we would like to go further. We would like to make a rough prediction of the size of the percentage of the group who, after comparable instruction, will have mastered the principle. Given the operational definitions set forth in Chapter IV, the interest is in roughly predicting the percentage who will meet the sufficient condition for mastery -- getting at least five correct out of the six items in the item group that corresponds to the principle. One cannot assume a normal distribution around the predicted mean, particularly when it is 4 or better, as one can see from working out a few comparisons between the mean difficulty indices and these percentages as they actually occurred on the testing that was done.

TABLE VI-5. Multiple Correlations, Cross Validation Estimates, and Standard Errors for Prediction Equations.

N=	Conditional 94						Class 123					
	Using Pre-Test			Without Pre-Test			Using Pre-Test			Without Pre-Test		
	Rm	Rxv	SE	Rm	Rxv	SE	Rm	Rxv	SE	Rm	Rxv	SE
Total Score	.81	.77	12.07	.73	.69	13.85	.87	.86	9.88	.81	.78	11.71
Component												
CF	.82	.78	5.17	.71	.66	6.31	.84	.82	4.71	.77	.74	5.51
SY	.72	.65	1.74	.67	.62	1.83	.78	.75	1.72	.76	.73	1.77
SU	.72	.67	1.69	.69	.64	1.74	.84	.82	1.62	.77	.74	1.91
Item Group												
1	.59	.47	1.05	.52	.41	1.10	.75	.71	.93	.59	.53	1.12
2	.77	.71	1.45	.65	.58	1.69	.71	.66	.84	.67	.62	.88
3	.59	.48	1.60	.59	.50	1.59	.76	.72	1.19	.66	.62	1.38
4	.63	.53	1.42	.51	.40	1.54	.60	.53	1.62	.54	.47	1.69
5	.61	.50	1.28	.46	.32	1.42	.63	.56	.89	.60	.54	.91
6	.56	.42	1.55	.40	.22	1.69	.73	.68	.94	.64	.59	1.05
7	.66	.58	1.81	.60	.53	1.91	.65	.59	1.61	.54	.46	1.77
8	.46	.26	1.48	.40	.21	1.51	.76	.73	1.28	.65	.60	1.49
9	.36	.00	1.55	.31	.00	1.56	.56	.47	1.37	.53	.46	1.39
10	.67	.59	1.44	.60	.53	1.53	.58	.50	1.40	.53	.45	1.45
11	.65	.56	1.46	.49	.37	1.66	.74	.70	1.11	.68	.64	1.21
12	.56	.43	1.76	.52	.42	1.79	.64	.57	1.04	.56	.49	1.11

Note: Cross validation indices estimated by means of the Lord-Nicholson formula. (Brogden, 1954, pp 377-400)

Rm is the multiple correlation

Rxv is the cross validation estimate

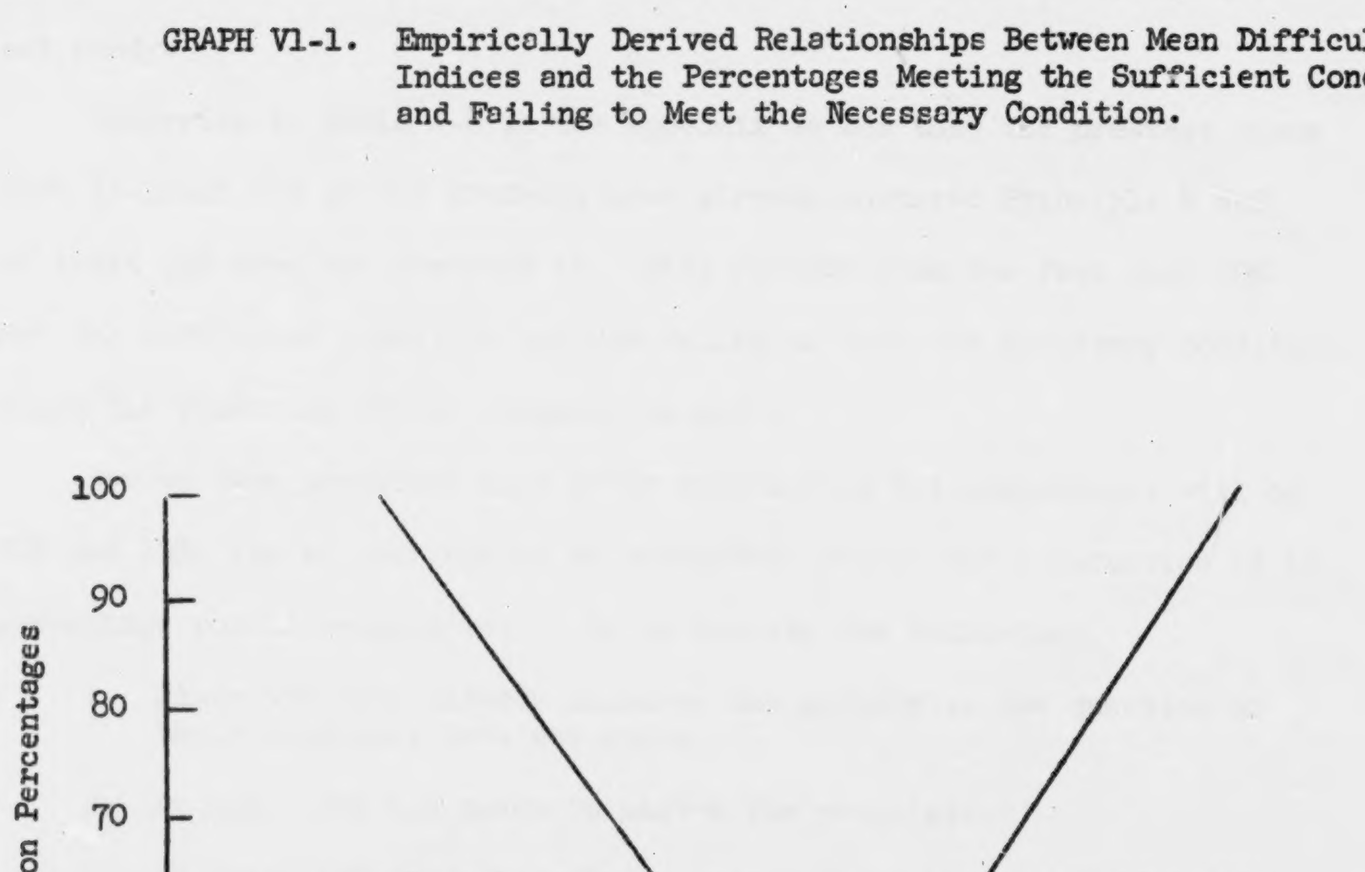
SE is the standard error.

So we plotted each mean difficulty index against each percentage meeting the sufficient condition on the pre- and post-tests for each of the two types of reasoning studied. These plotted points are reproduced in the Appendix as Graphs A-1 and A-3. A line of best fit was derived from these graphs and appears in Graph VI-1 in this chapter. A similar process was carried out for the percentages failing to meet the necessary conditions. See Graphs A-2 and A-4 in the Appendix. The line of best fit thus derived also appears on Graph VI-1.

Now we are in a position to complete the treatment of the example. Taking the predicted score of 4.91 we divide by 6 in order to secure the corresponding mean difficulty index. In this case it comes out 82%. Tracing this value up through two intersections, we obtain predicted percentages of 13% and 72%. The former is the percentage predicted to fail to meet the necessary condition and the latter is the percentage predicted to meet the sufficient condition. In other words we predict that somewhere around 72% of the students in our hypothetical case will have mastered Conditional Principle #4 and that somewhere around 13% will definitely not have mastered it. Incidentally the actual percentages for the 11th grade LDT's were 15% and 77% on the post-test.

Making use of the previous analysis of readiness for mastery and accepting the limitation imposed by our being able to provide only a given amount of a given cost of instruction, we can now exemplify the type of statement that we would like to make about the readiness of the members of a group to master a given principle of logic. Let us assume that these statements are to be made about the 11th grade LDT's before they received any deliberate instruction. What we want to do is to estimate the percentages

GRAPH V1-1. Empirically Derived Relationships Between Mean Difficulty Indices and the Percentages Meeting the Sufficient Condition and Failing to Meet the Necessary Condition.



who are ready to master a given principle (say Principle 4) and who are not ready.

Referring to Table A-8 in the Appendix we see that the pre-test shows that at least 50% of the students have already mastered Principle 4 and at least 23% have not mastered it. This follows from the fact that 50% met the sufficient condition and 23% failed to meet the necessary condition. About the remaining 27% no judgment is made.

Now we have predicted that after instruction the percentages will be 72% and 13%, for an increase of 22 percentage points and a reduction of 10 percentage points respectively. So we can say the following:

1. Since 50% have already mastered the principle, the question of their readiness does not arise.
2. At least 22% are ready to master the principle.
3. At least 13% are not ready to master the principle.
4. We do not want to be committed about the readiness of the other 15%. (100 -50 -22 -13 = 15)

Several qualifications need to be made, but for illustrative purposes we made the specific unqualified statements given above. Obviously with a group of the size of the 11th grade LDT's (N=26) one would not want to imply such precision. More rounding should occur. And such words as 'probably' and 'approximately' should be introduced.

This procedure for estimating the per cent of students who are ready to master a given principle is novel, and we do not know a way to estimate a confidence interval. The distribution is not normal. We can at this time only say that this procedure gives the best estimate that we can make on the basis of the data we have. The development of procedures for estimating a confidence interval is a task to which study needs to be given.

It should be noted that the procedure is based upon students who have taken the pre-test and thus have had the benefit of whatever learning accrues therefrom. If the equations without pre-test values are used, it should be kept in mind that part of the instruction given to those students was in the form of the administration of the pre-test.

c. Summary and Overview.

We have developed a method for making statements about the percentages of a group who are ready to master a principle and who are not ready. When pre-test scores are not available, the sufficient condition statements must be about percentages who either have mastered the principle or are ready to do so. The necessary condition statements would not be altered by the lack of these scores. One would still talk of the percentage who are not ready.

These equations can also be used to predict group mean total component scores. This is a more traditional use of such equations. The trouble is that there is as yet little operational interpretation of such scores.

E. FURTHER RESEARCH

The entire experiment should be replicated, but with more classes at each grade level to minimize the effect of class-wide factors. The intensive treatment used in this study leaves the results open to influence by idiosyncracies in the treatment. Furthermore the equations should be checked with students in other environments and with those in grades between those which were used in this study. The dividing line, if it exists, beyond which conditional reasoning can be effectively taught should be located more definitely.

Research of this sort should also be extended to lower ages and an attempt should be made to increase the amount of time devoted to teaching logic in order to see whether that would make much difference at the various age levels. The length of teaching time could also be used as a variable in the multiple regression equations.

The tests should be given on a pre- and/or post-test basis at various levels, including college, in order to develop a greater understanding of the meaning of predicted total and component scores.

Another set of multiple regression equations might be developed, making use of personality variables. These have been neglected in this study.

The possible anomalies between the class reasoning combination item groups 10 and 11 and the item groups representing their parts should be investigated and resolved if possible. One wonders why when there is improvement in the parts (and in ability to combine parts), there is not an improvement in the whole.

Last and perhaps most important, the possibility that verbal as opposed to conceptual learning accounts for the marked conditional-reasoning improvement among 11th graders must be investigated. This problem will require sophisticated indirect investigation.

F. CHAPTER SUMMARY.

The first part of this chapter was devoted to an analysis of the concept, readiness to master a principle. It was noted that the double dispositionality of the concept makes it difficult to justify empirically statements that make use of the concept. Another source of difficulty is its explosive-type dispositionality.

The result of the analysis goes roughly as follows: To say that Y is ready to master Principle X is to say that Y has the disposition, given suitable conditions, to develop the disposition to show, given suitable conditions, the sort of behavior that a person who has mastered Principle X would show. For simplicity's sake there is a circularity in the use of the term 'mastery' to define 'readiness for mastery', but this is resolved for present purposes by appeal to the operational definition of 'mastery' given in Chapter IV. The vagueness of the phrase 'suitable conditions' is unavoidable. The concept is vague in this respect.

After noting the dearth of literature dealing with the topic, capacity to learn logic, we described our experimental procedures, and said what we could about the nature of the teaching that was done.

Discussion of the results was broken up into two parts, the latter of which was the more speculative, because it attempts to make present-tense readiness statements. The first part was concerned with what our subjects actually did learn while under instruction. Thus it was essentially a discussion of their prior readiness for mastery and in effect made past-tense readiness statements.

There was a marked contrast between the learning of conditional reasoning that was effected and the learning of class reasoning. There was practically no learning of conditional reasoning in grades 5, 7, and 9; but in grade 11 there was a vast improvement. This improvement was concentrated mainly among the fallacy item groups, but also to some extent in those presenting contraposition and affirming the antecedent. For class reasoning on the other hand there were small amounts of improvement from grade 6 onward, the largest improvement under instruction being registered with the most complex items. This latter improvement perhaps indicates the efficacy of the instruction in handling and ordering complexities. Improvement in handling symbolic items was also noted.

One possible explanation of the marked difference between conditional and class reasoning is that the major learning that accounts for the vast improvement in conditional reasoning was the learning of the sufficient-condition meaning of the word 'if', which has no analogue in class reasoning. One difficulty with this explanation is that it leaves one wondering why the meaning of this word was not effectively taught in grades 5, 7, and 9.

On the basis of these results it seems that, given comparable students, class reasoning instruction might profitably be commenced at about age 12, and that conditional reasoning instruction might be commenced by age 16, possibly sooner. It was roughly at these ages that we found readiness for greater percentages of mastery.

The attempts to make present-tense statements about readiness for mastery of particular principles of logic were based upon the development of multiple regression equations which made use of the variables, grade, chronological age, IQ, socio-economic-status, and sex; and in addition the optional use of pre-test scores. The predicted scores are not reliable enough for use in individual cases, but a scheme was developed for making predictions of percentages of groups. These predictions, assuming the previous analysis of 'readiness to master Principle X', become statements about percentages who are ready to master the principle and who are not ready to master the principle. These statements about percentages assume comparable teaching treatment and thus the readiness statements generated are overprecise.

The major weakness of this study lies in the small number of subjects with which we worked (about 20-30 per grade level, with only 4 or 5 grade levels for each type of reasoning). This smallness was made necessary by the requirement that the treatment be intensive: that the instruction be carried out by people who knew logic, who knew how to teach at the level assigned and who had adequate time for class preparation and evaluation.

Chapter VII - The Past and Future of the Project

Although no date can be given to mark the beginning of the Cornell Critical Thinking Project, since it just grew Topsy-like, several events early in its existence can be dated. Two general critical thinking tests were prepared in 1961. A tentative theoretical grounding of the project was published in 1962 and has been under constant revision since then. A set of short reviews of works related to critical thinking was prepared and made available in 1962. These reviews were supported in part by a grant of the Ford Foundation to the Cornell Junior High School Project. In addition funds from the same source were used to pay for a collection of works related to critical thinking and a set of specimens of every critical thinking test that we could find. These two collections proved very helpful as research has progressed.

The Readiness Study: The Past.

In academic year 1961-1962 the Cooperative Research Branch of the U.S. Office of Education gave a grant to Cornell to study critical thinking readiness. That grant, which ran from May, 1962, to September, 1964, supported the research reported on here. It was felt that a readiness study would involve some of the basic work necessary before applied curriculum and method studies could be performed. This basic work included clarification of aspects of critical thinking, development of tests for aspects of critical thinking, and rough determination of the knowledge of, and capacity for learning aspects of critical thinking found in students of various levels.

The initial intent under this grant was to pick two aspects of critical thinking, probably deduction and assumption-finding, and perform the above-indicated basic work for each at all the levels of elementary and secondary school. We started with deduction, feeling that it, of all aspects of critical thinking, was the one most fully developed, and would be the easiest to handle. We hoped to make the study of deduction both a pilot study for the other aspects of critical thinking and a study of deduction in its own right.

As soon as we came to a detailed consideration of the nature and principles of deduction, we discovered not only that it as a field has not been comprehensively defined and categorized, but that the parts that have been worked on by logicians are not by any means clear and uncontroversial. We had to select among possible types of deduction and had to select within types. The content in the field of logic not being as well worked out as we expected, we had to devote a great deal of time to this job.

Although our original intent had been to teach the basic principles of deduction at each grade level, we discovered as a result of our analysis of logic that this would be too much to teach in the limited amount of time we were able to get the school system to commit to us, so we set about to find delimited parts of logic to teach. The plan became one of working on only one particular type of logic at each grade level. For the first and pilot year, we picked for study sentence reasoning, class reasoning, and ordinal reasoning. These are described in Chapter II.

Because of item and test tryout experience we assigned ordinal reasoning to the elementary grades only, since by the middle of the secondary school experience, most of the students whom we interviewed had already fairly well mastered the basic principles of ordinal reasoning under the present natural-cultural system. There seemed not much point to ask what basic principles of ordinal reasoning secondary students are ready to master when they have already generally mastered these principles.

During the first and pilot year of the study ordinal reasoning was studied in grades 1, 3, and 6; sentence reasoning in grades 2, 5, 7, 9, and 11; and class reasoning in grades 1, 4, 6, 8, 10, and 12. After extensive item and test tryouts, one test for each of the three types of reasoning was developed. It was used as a pre-test and a post-test for three typical classes at each grade level. Shortened versions and special techniques were used at the lower grade levels.

One of the three classes at each grade level was selected for intensive teaching of the type of reasoning assigned to the grade level. This teaching was done by one of our staff members, who were experienced teachers specially trained in logic. In each case it ran for one period per day for 10 instructional days (two weeks).

As a result of this pilot experience, we learned several important lessons:

1. That the amount of time devoted to the instruction was in general insufficient. We concluded this because we felt that we had not come anywhere near out students' limits for learning logic. This was a subjective impression, but quite probably valid.

2. That in future readiness studies, much more of a student's time must be devoted to the study of logic. Because we were both operating on a small scale and insistent upon control over what was being taught, we had to introduce our own staff members into the schools. This inevitably resulted in strong limitations by the schools on the amount of a student's time we could have. A possible way around this problem, a way not open to us this last year because of budget limitations, is to provide training in logic and the teaching of logic for a large number of teachers during a summer, and then make use of their regular classes during the entire school year.

3. That some test revision was still necessary.

4. That the first year's study should be treated only as a pilot study for the reasons given above and that the second year should consist of an improved study of readiness for learning deduction, with more of a student's time committed than during the first year.

5. That this improved study would require greater concentration of our resources and a consequent reduction in the scope of the effort.

6. That sentence reasoning would have to be cut down to conditional reasoning, omitting disjunction, alternation, and conjunction reasoning. This would be in order to reduce the number of principles to be taught,--that is, to make the teaching task more manageable. We judged the body of basic content in conditional reasoning to be roughly equal to that of class reasoning.

To achieve a further reduction we chose to eliminate the lower primary grades from the second year's study. The results of the first year indicated that, at least as we were going about teaching, not much learning of class and conditional reasoning was going on in those years. Although ordinal reasoning was learned in those years, ordinal reasoning is not as central in logic as class and conditional reasoning. Since something had to go, we dropped the years in which the most central parts of logic, given our limited amount of teaching, were not getting across to the students.

The concentration of resources was achieved by increasing the amount of teaching time to 15 instructional days on logic, generally accompanied by an additional 5 days in which our staff member devoted himself to the advancement of the subject matter which was being replaced by the logic instruction.

The second year's study is that described in the body of this report.

The Tests.

The original plan called for one deductive logic test, consisting of six items per grade level of about 50% difficulty for the given grade level (with some extras at both ends). The result was to have been a 90-item test modeled after Burt's Graded Reasoning Test (1919).

As indicated earlier one of the results of the analysis of logic was the decision to split deductive logic into parts in order to make the teaching more manageable. Hence one test of logic in general would no longer suffice. But there was another reason for changing the original test plan. The analysis of logic showed not only that there exists a small set of basic principles for each of the types of logic, but that a general deductive test would yield a score devoid of intrinsic meaning. We felt we could refine the measure sufficiently to have it be in terms of specific principles of logic. Scores on these principles together with the operational definitions of mastery of the principles, which are given in Chapter IV, yield a score which hopefully has meaning which is not as arbitrary as a total score on a jumbled array of assorted deduction items.

Hence instead of one deduction test which yields scores only in terms of norms, we developed a test for each of the types of logic studied which would yield scores on the principles of logic, scores which are interpretable in ordinary language.

The original plan called for 90 items. There are 144 items in the class and conditional reasoning tests (72 items apiece). An additional 66 items are in the ordinal reasoning test, which is not reported on in this study. It is being held in reserve for a future phase of the study. In the construction of the three tests, an original pool of 740 items was prepared -- and supplemented as test development, interviews and tryouts were conducted. Approximately 30 students were individually interviewed in the preparation of the tests, and about 500 students were used in various stages of tryouts.

The Readiness Study: The Future

This report covers only the first of a number of phases in the study of critical thinking readiness in grades 1-12. In the tentative designations of phases to follow, we have broken the process of doing a readiness study of an aspect of critical thinking into three parts: the analysis of the aspect; the development of a test (or tests) for that aspect; and the combined process of testing, teaching, and developing readiness equations. All three parts would in general take about three years to carry out for each aspect of critical thinking. Starting one aspect per year would result in an overall time commitment of about 10 more years.

Table VII-1. A Tentative Time Table for Future Phases of the Critical Thinking Readiness Study.

Academic Year	Aspect Analysis	Test Development	Testing, Teaching, and Readiness Equations
1967-68	Assumption-Finding	Ordinal Reasoning (revision) Class and Conditional Reasoning (Grades 1-3, adaptation)	Ordinal Reasoning: Grades 1-6 Class and Conditional Reasoning: Grades 1-3
1968-69	Generalizing	Alternation and Disjunction Reasoning Assumption-Finding	Alternation and Disjunction Reasoning: Grades 1-12
1969-70	Reliability of Observation Statements	Generalizing	Assumption-Finding: Grades 1-12
1970-71	Reliability of Authorities	Reliability of Observation Statements	Generalizing: Grades 1-12
1971-72	Hypothesis Warrant	Reliability of Authorities	Reliability of Observation Statements: Grades 1-12
1972-73	Detecting Ambiguity	Hypothesis Warrant	Reliability of Authorities: Grades 1-12
1973-74	Overvagueness and Overspecificity	Detecting Ambiguity	Hypothesis Warrant: Grades 1-12
1974-75	Theory Warrant	Overvagueness and Overspecificity	Detecting Ambiguity: Grades 1-12
1975-76	Instead of aspect analysis and new test development, this portion will be devoted to repeating some investigations, and writing a total report.	Theory Warrant	Overvagueness and Overspecificity: Grades 1-12
1976-77			Theory Warrant: Perhaps Grades 13-16

CHAPTER VIII - Summary

In this chapter we shall try to indicate the nature of the problems that we set for ourselves, the interpretation of our key concepts, the procedures used to pursue our objectives, and our results. Many qualifications will be omitted in order that this overview be in condensed form. The reader is referred to the specific chapters in the body of this report for a fuller treatment of each topic. This particular chapter is organized in the same manner as the report as a whole and will generally use chapter headings as sub-headings.

Introduction: The Statement of the Problems.

This study is concerned with two general empirical questions, which provide the focus of the study, and several conceptual ones, the answers to which, although auxiliary with respect to the empirical questions, are important for the clarity and understanding they provide in dealing with the empirical questions. The empirical questions are 1) that of readiness for mastery of logic, and 2) that of the natural-cultural development of mastery of logic.

The readiness question is concerned with the level (age, grade, etc.) at which and the extent to which students are ready to master logic. The natural-cultural development question is concerned with the levels at which and the extent to which logic is mastered by students who have not been exposed to deliberate instruction in logic. Presumably this development is attributable to both natural and cultural factors; hence the title: "Natural-Cultural Development of Logic".

These two questions are closely related. If we find out that a certain principle of logic has already been mastered by someone as a result of natural-cultural processes, then the question of readiness for mastery of the principle does not arise. Furthermore the process of estimating what a student is ready to master is facilitated by knowing the degree of mastery at a given moment. In investigating the readiness question, which was our primary interest, we unavoidably gathered information on the natural-cultural development question, which is the subject of considerable other research, especially that of Piaget. Both questions are discussed in this report.

In order to discuss these questions some conceptual problems had to be dealt with first. First of all we had to delimit the area of logic and select a concept of a valid argument that is in agreement with intuition and is relatively non-controversial. Next, we had to develop operational definitions of 'mastery of a principle of logic'; but in order to do this we also had to do some work on the concept of an operational definition. Lastly the concept of readiness required analysis.

It might well be that the actual contribution made by this study is greater in the area of conceptual clarification than in the actual empirical findings. The empirical findings are limited by the geographical and numerical limitations of the subjects with whom we worked. But the conceptual suggestions that were developed can be applied to other studies, of which many are needed.

In order that we can assume the understanding of the conceptual points in the presentation of the empirical findings, the conceptual points will in this chapter precede the empirical findings which depend upon them.

The Nature of the Subject-Matter: Logic.

The subject matter, the mastery of which is the subject of this study, is logic, which itself is an important part of our basic interest, critical thinking. It is that part of critical thinking which deals with whether a conclusion follows necessarily from the premises that are offered in support of it. The centrality of this part of critical thinking has been argued for elsewhere (Ennis, 1962).

Logic is not concerned with whether the premises or conclusion are true, but simply with whether the former necessarily implies the latter.

Two Types of Logic.

There are many different types of statements between which this relationship of necessary implication can hold. Roughly speaking, the field of logic is categorized according to the types of statements treated.

Of the many types of logic which exist (no one has successfully defined them all), we have selected two of the most common, conditional logic and class logic.

Conditional logic deals with statements containing the words, 'if', 'only if', and/or their synonyms. It is called "conditional logic" because the words just mentioned are frequently used to introduce conditions upon which the truth of the rest of a statement containing them depends. The part of a statement introduced by the word 'if' is called the "antecedent". The rest (except for the word 'then', when used to introduce the rest) is called the "consequent". A simple argument in conditional logic proceeds by presenting a conditional statement, then affirming or denying either the antecedent or the consequent, and drawing as a conclusion either the affirmation or denial of the consequent or antecedent respectively. If the conclusion follows necessarily, then the argument is called valid. If the conclusion does not necessarily follow, then the argument is called invalid.

Here is an example of a valid argument in conditional logic, which has been adapted from the test used in this study:

Premises:

If the car in the parking lot is Mr. Smith's, then it is blue.
The car in the parking lot is not blue.

Conclusion:

The car in the parking lot is not Mr. Smith's.

Class logic, which is also concerned with the validity of arguments, deals with statements which have a different structure and a different relationship to each other. In conditional logic, the antecedent and consequent appear and reappear in the argument essentially unchanged, except for the possible negation of one or the other. Statements in class logic have as their basic interchangeable units not sentences that can stand independently of each other, but terms and phrases which refer to individuals or groups. For example, the statement, "All the cars in the garage are Mr. Smith's cars", is a class logic statement because the basic units in the statement are the terms, "the cars in the garage" and "Mr. Smith's cars", which refer to groups. The statement gives a relationship between these groups.

Here, for example, is a valid argument, which is an adaptation of an item from one of the tests that we developed:

Premises:

All the cars in the garage are Mr. Smith's cars.
All Mr. Smith's cars are Fords.

Conclusion:

All the cars in the garage are Fords.

Principles of Logic. For purposes of instruction and testing we selected what seemed to be the basic elementary principles in each of the above two types of logic. They are listed below and can be found exemplified in Chapter II:

Conditional Logic:

1. Given an if-then sentence, the affirmation of the if-part implies the affirmation of the then-part.
2. Given an if-then sentence, the denial of the if-part does not by itself (as a result of its being an if-part) imply the denial of the then-part.
3. Given an if-then sentence, the affirmation of the then-part does not by itself (as a result of its being a then-part) imply the affirmation of the if-part.
4. Given an if-then sentence, the denial of the then-part implies the denial of the if-part.
5. The if-then relationship is transitive.
6. An if-then sentence implies its contrapositive.
7. The if-then relation is non-symmetric.
8. Given an only-if sentence, the denial of the only-if part implies the denial of the major part.
9. Given an only-if sentence, the affirmation of the major part implies the affirmation of the only-if part.
10. The denial or affirmation of one part of an if-and-only-if statement implies respectively the denial or affirmation of the other part.
11. Given an only-if sentence, the affirmation of the only-if part does not by itself (as a result of its being an only-if part) imply the affirmation of the major part.
12. Given an only-if sentence, the denial of the major part does not by itself (as a result of its being the major part) imply the denial of the only-if part.

Class Logic:

1. Whatever is a member of a class is not a non-member of that class and vice versa.

2. Whatever is a member of a class is also a member of a class in which the first is included. (This implies that class inclusion is transitive.)

3. Whatever is a member of a class is not (as a result of that relationship) necessarily a member of a class included in that class.

4. Class exclusion is symmetric.

5. Whatever is a member of a class is not a member of a class excluded from the first.

6. Whatever is not a member of a class is not (as a result of that relationship) necessarily also not a member of a class in which the first is included.

7. Whatever is not a member of a class is not (as a result of that relationship) necessarily a member of (nor a non-member of) another class which is excluded from the first.

8. Whatever is not a member of a class is also not a member of any class included in the first.

For purposes of discussion of development of knowledge and readiness, we have grouped most of these principles according to the following headings: conversion, negation of antecedent, contraposition, transitivity, and 'only-if' principles. Although the terms used in this list of principle groups are primarily logical terms, decisions about categorization of principles are for psychological reasons as well. This is a rough classification system. The groupings and descriptions are inexact but useful.

Conversion: The converse of a statement, which, roughly speaking, is the statement turned around with the beginning and end exchanged, does not necessarily follow from the original statement. The conversion principles are Conditional Principles 3 and 7, and Class Principle 3. We call them "fallacy principles", because they indicate logical fallacies.

Negation of Antecedent: This is also an invalid move. It consists of negating the first part of a statement and on the basis of that negation, concluding that the second part must be negated as well. Conditional Principle

2 and Class Principles 6 and 7 are negation-of-antecedent principles, and thus are also fallacy principles.

Contraposition: This is a valid move. It consists in negating the second part of a statement and on that basis concluding that the first part should be negated also. Conditional Principles 4 and 6 are contraposition principles, as are Class Principles 5 and 8.

Transitivity: This too is a valid move for the types of logic we studied. It is best explained by a symbolic example: If X has a given transitive relationship to Y, and Y has that same transitive relationship to Z, then X has that relationship to Z. Being taller than is a transitive relationship, for example. A transitive relationship is one, so to speak, that carries through an intermediary when an intermediary exists. Conditional Principle 5 and Class Principle 2 are clear cases of transitivity principles. Conditional Principle 1, is like a transitivity principle, even though it does not quite fit the symbolic form mentioned above. It is similar in that the affirmation of the antecedent is carried through to the consequent.

'Only-if' principles: These are conditional principles which we have grouped together because of the occurrence of the phrase 'only if' in arguments to which they apply. The phrase 'only if' indicates a necessary, but not necessarily sufficient condition for the truth of the rest of the sentence. Three of them indicate valid arguments and two indicate invalid arguments. The principles are 8, 9, 10, 11, and 12. We group all together because although there are important differences, they seemed to us to be conveniently grouped for teaching.

An intensive study of Piaget's logic was conducted for the purpose of comparison with the types of logic presented above. Basically we found the logics similar although we had reservations. We concluded that Piaget's conclusions and our conclusions are therefore relevant to each other.

Because of the large amount to be learned in each of the two types of logic we studied, we assigned these two types of logic to alternate grade levels within the range studied. Conditional logic was tested for and taught at grade levels 5, 7, 9, and 11, while class logic was tested for and taught at grade levels 4, 6, 8, 10, and 12.

Basic Data on Subjects.

Although we had 803 subjects distributed in grades 4-12 (inclusive), not all were used in the readiness part of the study. All were used in the natural-cultural development part of the study. At each grade level one class-size group was selected for purposes of instruction in logic. There were 217 of these subjects, called the "LDT's" (for 'Logic Deliberately Taught) who by design were intended to be fairly representative of the upper New York State school system from which they were drawn.

From the same school system comparable classes were selected, again one at each grade level. These subjects, called the "LNDT-1's" (for 'Logic Not Deliberately Taught, 1st group'), were 211 in number.

From another upper New York State school system another 375 students, called the "LNDT-2's", were selected for use, together with the LDT's and the LNDT-1's, in the natural-cultural development part of the study.

The following table gives some gross figures about these subjects. "SES" stands for socio-economic status, and is based upon a seven-point occupational rating scale developed by Warner (1949, pp140-141). 1 is high and 7 is low. IQ scores for LDT's and LNDT-1's are Lorge-Thorndikes; for LNDT-2's, predominately the California Test of Mental Maturity. Further details about the instruments and data-gathering procedures can be found in Chapter III.

Table VIII-1 Gross Basic Data on Subjects.

	N	Mean IQ	SES
LDT's	217	116.8	3.8
LNDT-1's	211	116.3	3.5
LNDT-2's	375	108.4	None Calculated
Total	803	112.7	3.6 (LDT's and LNDT-1's only)

The Cornell Deduction Tests.

Two deduction tests were developed, one for each of the types of logic treated. They are called "The Cornell Conditional-Reasoning Test, Form X" and "The Cornell Class-Reasoning Test, Form X". These tests were constructed with the purpose of determining whether all but one of the principles of logic listed earlier are mastered or alternatively, the degree to which they are mastered. The one, Conditional Principle 12, was omitted in order that combinations of principles could be included without lengthening the conditional test. Each test contained 72 items in 12 item groups of 6 items apiece. Each group of 6 items embodies a principle or combination of principles. The six items within any one group were scattered. The tests were deliberately kept short in order that they could be administered in a 40-minute period.

Operational definitions of mastery of a principle were tied to the item groups. Getting at least 5 of the 6 items correct was deemed to be a sufficient condition for probable mastery of the principle and getting at least 4 correct was judged to be a probable necessary condition. (Thus getting three or fewer correct implied lack of mastery.) Formally put, the following are the operational definitions of 'mastery of Conditional Principle 4':

> If Y is given "The Cornell Conditional-Reasoning Test, Form X" under standard conditions; then if Y answers correctly at least five of items 8, 16, 22, 29, 35, and 39, Y has probably mastered Conditional Principle 4.
>
> If Y is given "The Cornell Conditional-Reasoning Test, Form X" under standard conditions; then it is probable that Y has mastered Conditional Principle 4 only if Y answers correctly at least four of these items: 8, 16, 22, 29, 35, and 39.*

Making use of the distinctions among types of content made by Wilkins (1928) we included three content components in the tests: the concrete familiar, the symbolic, and the suggestive components. Each item group contains four concrete familiar items, one symbolic item, and one suggestive item.

Concrete familiar items are as the name suggests; furthermore, there is no reason to believe that a subject will, because of background factual knowledge, accept or reject the conclusion of concrete familiar items. The previous sample arguments about cars have concrete familiar content. They are about familiar concrete things, but there is no reason to think that a student will believe or disbelieve that all of Mr. Smith's cars are Fords.

The symbolic component items used symbols instead of words referring to objects in the sentences in the argument. And the suggestive component items are such that the validity status of the argument differs from the truth status of the conclusion, which truth status is presumed to be well-known to the subjects. For example if the conclusion were "All cats are black", which presumably is known to be false, then the argument would be

* The theory and defense of such definitions have been presented elsewhere by the principal investigator (Ennis, 1964).

either valid or such that it does not imply the denial of the conclusion, and would have to be recognized as such. This example is the conclusion of class item 31, which contains a valid argument, thus requiring an answer of "Yes."

The directions do not vaguely ask for a judgment about the conclusion. They ask whether, on the assumption that nothing but the premises are known, the conclusion would be true. Six sample problems are used to illustrate this type of question.

Reliability. Reliability was estimated by means of test-retest correlations at each grade level for the LNDT-1's and LNDT-2's combined. There was approximately a ten-week interval between test and retest. The mean of total score correlations was .75 on the conditional reasoning test and .83 on the class reasoning test.

Component score reliability estimates are lower. Mean concrete neutral test-retest correlations were .65 for conditional reasoning and .79 for class reasoning. Symbolic component mean test-retest correlations were .53 and .50 respectively. For the suggestive component the means were .55 and .63 respectively.

As might be expected the reliability estimates for the item groups were again lower. Means were .52 for conditional and .50 for class reasoning.

Validity. Because of the lack of a dependable outside criterion, no concurrent or predictive validity estimates were computed. Basically the type of validity with which we were concerned was construct validity. Much of the argument for construct validity here depends on the content analysis on which the test was based. A study of reasoning in newspaper editorials, an auto mechanics handbook, and two U.S. Supreme Court decision opinions, together with intensive study of logic and consultation with logicians, provided the basis for the content analysis judgments.

A series of tryouts, concerned with both content and form of the items was conducted. The finished product was the result of eighteen months of concentrated effort on the nature and types of logic, item format, directions, and content. An inspection of the items reveals them to be clear applications of the principles of logic presented earlier. The details of item assignment are to be found in Chapter IV.

The rest of the argument for construct validity is to be found in three areas: correlations with familiar measures, item analysis, and the role played by this test in the current study. Correlations with familiar measures, which are mainly based upon pre-tests given to LDT's and LNDT-1's, may be found in detail in Chapter IV; 423 subjects were used for these correlations, 188 on the conditional reasoning test, and 235 on the class reasoning test. Some of the more important ones follow.

Mean correlations with chronological age at a given grade level were -.11 and -.12 in conditional and class reasoning respectively, which relationships are not statistically significant.* In other words, at a given grade level, given the present system of advancement through the grades, there is if anything, only a slight negative relationship between age and mastery of logic. When the various grades are grouped together, then, as might be expected, there is a clear positive relationship between age and logic mastery, the correlations being .58 and .68 for the two tests respectively.** Both are statistically significant.

Mean correlations with (Lorge-Thorndike) IQ were .58 and .52 respectively, which are statistically significant and are about what one tends to find in correlations between subject matter tests and IQ. Correlations with the occupationally- based index of socio-economic status averaged .20 and .26 on the two tests respectively, both of which correlations again are statistically significant.

---

* All significance tests used the 5% level as the criterion.

** These particular correlations were based upon a sample of the LDT's; conditional reasoning N = 64; class reasoning, N = 82.

There appears to be no relationship between mastery of logic (among students not yet exposed to instruction in logic) and sex. The mean correlations were .10 and .00 respectively and were not statistically significant. If a relationship does exist, it presumably is a very weak one.

The above relationships between these two tests and common measures contribute to the argument for construct validity insofar as they make sense and fit into some sort of theory (though it be at a low level). That there be either no relationship or a slight negative relationship between knowledge of logic and chronological age within a given grade is not surprising, given that promotion is not a completely automatic thing. If bright young students can at least occasionally move ahead, on the assumption that knowledge of logic is related to brightness, then a positive relationship between chronological age and knowledge of logic at a given grade level would be defeated.

If knowledge of logic can be presumed to increase with age, then there should be a positive correlation between logic test scores and chronological age, when separate grades are grouped together. And if logic is an intellectual ability, then scores on these logic tests should be related to IQ scores. On the same assumption, with the additional assumption that there is greater intellectual development among the children of the higher socio-economic classes (excluding upper-upper classes, of whom there were few or none in this study) at least some positive relationship between socio-economic status and logic test score should be found. And lastly we know of no reason to expect one sex to be better than another on a test in logic.

The results on these tests conformed to expectations and are understandable and plausible. Hence these correlations with familiar measure support the argument for construct validity. Naturally this is only support, not confirmation, since other interpretations could be given to the data.

Item analysis information, which is presented in detail in Chapter IV and the Appendix, gives us more information about the test. We have no theory to predict the results of the item analyses that we performed, but do not find the results exceptional.

The mean difficulty levels of the total tests (ranges: 47.5* - 61.5 for conditional; 49.3 - 72.7 for class) show that the tests are not too difficult for the grades at which they were administered -- if one is interested in total score. Since our major interest is in whether or not a principle has been mastered, this criterion is not one that must be satisfied in order that the test be usuable for our purposes. That it is satisfied is interesting, but not crucial.

The mean discrimination indices were 26.0 on the conditional test and 29.8 on the class test. When the mean discrimination indices were computed for the various item groups at the various grade levels, some variability was evident, which will be discussed under the next topic. There was one group which consistently failed to discriminate, Conditional Item Group 12, which calls for the combined application of Conditional Principles I and II. This lack of discriminating power we attribute to the great difficulty of the items in this group (index hovering around 20% for all grade levels). Since the function of the test is to see if certain principles and combinations thereof are mastered, the lack of discriminating power of this item group is not a defect. It would be a defect if the test were used with similar groups for purposes of discriminating among the members of those groups.

---

* These figures are means of the percentages of students getting each item right at a given grade level.

The last type of evidence regarding construct validity is the role played by these tests in this and other studies. Since they have not yet been used in other studies, only the results of this one can be presented. The rest of this chapter, in its description and analysis of the results, is thus also an implicit discussion of construct validity, on the assumption that greater understanding of test results contributes to construct validity.

The Natural Cultural Development of Knowledge of Logic.

Partly because Piaget's claims about development (without deliberate instruction) of knowledge of logic are vague and sometimes ambiguous, and partly because we worked with students in the age range 10-18, there is in this study actually no direct and unequivocal test of his views. But his studies did suggest the following questions:

1. Is there actually a development of logical ability as children grow older?
2. Does this development (if there is any) come in stages?
3. Is conditional logic mastered by age 11-12?
4. Is class logic mastered by age 11-12?
5. Is the truth-validity characteristic (essentially the mastery of the suggestive component) achieved by age 11-12?

6. Within each type of logic, is there a development of one sort of thing before another such that some things are more difficult than others at a given level?

Answers to the first five of these questions are suggested or indicated in the literature, are suggested by our study, and are fairly obvious and routine for anyone who has worked with children and thought about the questions. The answers are as follows:

1. Yes.
2. That depends on what you count as a stage, but gross plateaus extending over a period of years were not evident.
3. Partly.
4. Partly.
5. Partly.

The sixth question is an extension of the 3rd, 4th, and 5th in that it is concerned with the extent to which various parts are mastered. It is the sixth question which is the interesting one to our minds and is the one toward which this study has made contribution, if it has made any contribution in the area of the natural-cultural development of logic. Part of the contribution lies in the formulation of the question, and part lies in the empirical findings. The latter part is inevitably quite modest because of the size of the group and the fact that it is composed of students from only one small part of the world.

We found considerable similarity in the developmental patterns of the two types of logic studied, with the exception that class logic appeared to be easier all along the line. The principles expressing the basic fallacies (conversion, negating the antecedent) were the most difficult at ages 10-12, but there was considerable improvement in these over the period studied (age 10-18). Among the validity principles, the contraposition ones contrast with the transitivity principles in that although both are at medium difficulty at age 10-12, there is little improvement over the years in the contraposition principles, whereas there is considerable improvement in the transitivity principles.

Although we found parallel patterns among the types of principles in the two types of reasoning studied, we did not find as much parallelism between the two types of reasoning when we examined the three components of reasoning (concrete familiar, symbolic, suggestive) which were built into the tests. One parallel feature that we did find was the fairly regular improvement over the years studied in all three components on both tests. But there was a marked difference.

On the conditional reasoning test the three components were of about equal difficulty at each grade level. But on the class reasoning test, the concrete familiar component was in general easier than each of the other components, particularly the symbolic component. At all five grade levels the mean difficulty index differences between the concrete familiar component and the symbolic component exceeded ten percentage points, and in three of the five grade levels the differences exceeded twenty percentage points and were statistically significant. And the differences between the concrete familiar components and the suggestive component also exceeded ten percentage points at all grade levels tested. One of these differences was statistically significant.

Previous studies that have shown the suggestive and symbolic components to be more difficult than the concrete familiar component have all used class reasoning. One wonders why the difference in difficulty did not appear in conditional reasoning. In developmental terms one wonders why the suggestive and symbolic components lag behind the concrete familiar component in class reasoning, but not in conditional reasoning. Explanations which were proposed in Chapter V suggest that the differences result from the nature of the logic being investigated and from the difficulties that uninstructed students have in dealing with the use of symbols to represent sentences.

The Development of Readiness to Master Logic.

The concept *readiness to master a principle* was studied and the following rough and somewhat vague analysis was the result: To say that Y is ready to master Principle X is to say that Y has the disposition, given suitable conditions, to develop another disposition, which is the disposition to show, given suitable conditions, the sort of behavior that a person who has mastered Principle X would show. This analysis assumes a prior specification of the meaning of 'mastery of Principle X', which is in part provided by the operational definitions given earlier. The analysis is complicated by the double dispositionality of the concept; a disposition to develop a disposition is under consideration. And the analysis twice makes use of the vague terms 'suitable conditions'. But this is because the concept being analyzed is vague in the same way. The person using the concept *readiness* must judge whether the amount of effort needed to achieve the desired mastery or learning is worth the trouble. And he makes **implicit** reference to possible means for achieving the mastery or learning but he need not specify the means. A person claiming that Y is ready to master Principle X implies that the means of bringing Y to this mastery exist and are feasible, but the means are not necessarily specified in the statement claiming readiness. Hence the concept, like the phrase, 'suitable conditions', is vague.

In this part of the study, we concentrated on two empirical questions in addition to the question of the analysis of readiness for mastery:

1. What were our subjects ready to master?
2. What are fairly comparable students ready to master?

The first question is a question of description and the second a question of prediction.

In the first question the term 'our subjects' refers to the LDT's. They were given the pre-test, were taught logic for 15 days, one period per day, and approximately six weeks later, were given the post-test, which was the same test as was used for the pre-test. Teaching was done by members of the project staff, who were given special training in logic and were experienced teachers at the grade levels with which they worked.

1. What Our Subjects Were Ready to Master.

The parallelism between the two types of logic that we found in the natural-cultural development of principles was not to be found in the teachability of principles - nor in improvements in component and total scores. In class logic modest improvements in total scores, component scores, and principle scores generally occurred in students from age 11-12 upward during the teaching period. But in conditional logic the younger students in our study registered virtually no improvement, while the upper secondary school students registered a marked improvement in total scores, component scores, and the fallacy principle scores after having been taught.

Class reasoning is apparently teachable to some extent from age 11-12 onwards. Students younger than that did not benefit from the 15 days of instruction that we were able to give them. Perhaps under different conditions - or with more time - they also would have benefited. From 11-12 onward there is apparently modest fairly even improvement as a result of whatever natural-cultural sources are operating, and deliberate teaching of the sort we did can contribute modestly to this improvement. By age 17-18 there was as a result of existing natural-cultural influences on our LDT's considerable mastery of the basic principles of class logic. Our teaching made a modest improvement. Overall, talking in terms of readiness, we can say that from age 11-12 onward our students were ready for modest improvements in mastery of the principles of class reasoning, and that by age 17-18, the group as a whole was ready to make the modest improvement that when made justifies our saying that for practical purposes they have mastered the basic principles of class logic.

Conditional logic makes a different story. Apparently, when given the sort of instruction we provided, our LDT's were not ready to make much improvement until upper secondary; but by age 16-17 were ready to make great strides. These great improvements in mastery were particularly evident among the fallacy principles (where there was much room for improvement); but they also occurred among the contraposition principles, the affirming-the-antecedent principle and to a slight extent the transitivity principles. No improvement was registered among the 'only-if' principles, though this might be because insufficient time was devoted to them and at the outset the 16-17 year olds were fairly good at them.

It is quite possible that our data suggest a line between the ready and nonready that is sharper than it should be. We have some question about the motivation of the 9th grade LDT's. In any case our 11th graders were by and large ready to master the fallacy principles, and our 5th and 7th graders were not. This is the most striking finding in this readiness study.

2. That Which Fairly Comparable Students Are Ready to Master. Using the relationships between scores on certain variables (described earlier) and the LDT post-test scores, and the interrelationships among these variables, we constructed multiple-regression equations for the purpose of predicting what other fairly comparable students are ready to master. Two sets of equations were prepared, one making use of pre-test scores on the same logic test, and the other not doing so. Variables used in both sets are grade level, chronological age, IQ, socio-economic status, and sex. All grade levels on each test were combined; that is grades 5, 7, 9, and 11 (94 students in all) were combined for the conditional logic equations, and grades, 4, 6, 8, 10, and 12 (123 students in all) were combined for the class logic equations. Equations for each item-group score, each component score, and each total score were prepared. Altogether that makes 64 equations.*

Total score cross validation estimates with and without the use of the pre-test were .77 and .69 respectively for conditional logic and .86 and .78 respectively for class logic. Component score cross validation estimates ranged around .70 for conditional logic and around .75 for class logic, with the ones using pre-test score running higher than the ones without. Conditional logic item group cross validation estimates ranged around .50 and .40 respectively with and without pre-tests; class logic estimates ranged around .60 and .55 respectively.

These item-group cross validation estimates are not adequate for the prediction of individuals scores. Whether they are high enough for prediction of central tendencies of groups is an open question, but because of our interest in working out a technique for predicting mastery in a group, we developed such a technique and offer it as one which must be refined and investigated, both empirically and statistically.

---

* (12 + 3 + 1) (2) (2). Not all item groups uniquely tested for a principle. There were twelve item groups in each test and an equation constructed for each one.

The technique is as follows: Insert into the equation the mean values of the variables for the group in which we have an interest. Extract a predicted mean item group score. Convert the predicted score into an item group mean difficulty index. Use this to enter a graph which provides an empirically derived conversion to percentages meeting the sufficient condition for mastery and failing to meet the necessary condition for mastery. (See operational definitions on p VIII-10.) The results are the percentages that are predicted to obtain after instruction. If the pre-test has been given, then one can readily obtain the percentage that already have mastered the principle, and estimates of the percentage that are ready to master and the percentage that are not ready to master it.

Weights and constants for constructing these equations can be found in the Appendix in Tables A-11 through A-14. The graph for converting from mean difficulty indices to necessary and sufficient condition percentages is to be found in Chapter VI (Graph VI-1). The following is a sample working through of the above-described process, making use of the mean value of the 11th grade LDT's for purposes of illustration. Mastery of Conditional Principle #4 is the subject of this prediction effort:

Values going into the equation:

	Measured Value (rounded)	Weight
Grade	11	-.001
Chronological Age	199.8 mos.	.0074
IQ	116.0	.0064
Socio-economic Status	3.5	-.127
Sex (male, 1 female, 2)	1.5	.423
Total Score on Pre-Test	63.7	.0384
Pre-test Score on this Item Group	4.26	.314

Constant: -1.26

Predicted score = (-.001) (11) + (.0074) (199.8) + (.0064) (116.0) + (-.127) (3.5) + (.423) (1.5) + (.0384) (63.7) + (.314) (4.26) - 1.26 = -.01 + 1.48 + .74 - .45 + .63 + 2.44 + 1.34 - 1.26 = 4.91, predicted score.

This figure, 4.91, can be rounded or not, depending on what one is going to do with it.

The predicted score of 4.91 corresponds to a mean difficulty index for this item group of 82% (4.91/6 = approx. .82). Entering graph VI-1 we obtain predicted percentages of 13% failing to meet the necessary condition and 72% meeting the sufficient condition for mastery. Incidentally the actual percentages turned out to be 15% and 77%.

On the pre-test 23% failed to meet the necessary condition and 50% met the sufficient condition. Hence the claim that results from the prediction is that probably at least 22% (72% minus 50%) are ready to master the principle and at least 13% are not ready to master it. 50% had already mastered it; the other 15% (100--50 -22 -13 = 15) make up the group about which we do not want to make any sort of commitment.

Needed Further Research.

This study has only begun to explore the topic. The following steps are desirable:

1. The checking of the response form (multiple choice) of the tests against an open-ended test form.

2. Revision of the tests to adapt them to students in the first three grades.

3. Lengthening of the tests so that measurement of mastery of each principle by an individual becomes more reliable.

4. Replication with more classes at each grade level in different types of environment to minimize the effect of class-wide factors. The in-between grade levels should also be used. This should be done for both the natural-cultural and readiness parts of the study.

5. Extension with adaptation to lower age levels (ages 6-10), again for both empirical parts of the study.

6. The use of variation in amount of teaching time as a variable in the multiple regression equations.

7. More radically, the training in logic of full-time teachers, followed by their instructing logic in their own classes, in order to secure more time devoted to logic teaching in LDT classes, and at the same to maintain a more realistic situation. Again the readiness questions would be asked.

8. A close examination of the source of improvement in conditional logic among the LDT upper secondary students, with this specific question in mind: Is this learning merely the acquisition of new meanings for words or is it an increase in grasp of concepts and principles?

9. Empirical and mathematical investigation of techniques for predicting percentages who are ready to master a principle, making use of the operational criteria advanced here (or similar criteria). Such investigation should attempt to indicate the extent to which such procedures are likely to be in error.

10. The investigation of the readiness development and natural-cultural development of other aspects of critical thinking, including analysis of the aspect, test development, and testing and teaching of the aspect. It would probably be better in the future if existing classroom teachers were used, but only after thorough training.

Overview.

The products of this study are as follows:

1. The specification of a set of basic principles of two major types of logic, conditional logic and class logic.
2. Two logic tests, one on each of these types of logic.
3. A theory of operational definitions.
4. Operational definitions of mastery of each of the principles in terms of the two tests.
5. An analysis of Piaget's conception of logic.
6. A suggestion of the patterns of natural-cultural development of knowledge of logic from ages 10-18 among students like those studied here.
7. An analysis of the concept, readiness to master a principle.
8. A description of what amounts of logic our subjects were ready to master (given 15 days of logic instruction).
9. A set of 64 multiple regression equations for predicting the amount of mastery of logic (given comparable instruction) in groups comparable to those with which we worked.
10. A procedure to estimate the percentage of students in a given class who are ready to master and the percentage who are not ready to master a given principle of logic.

Further research is needed on all these items.

CITED REFERENCES

Bonser, Frederick G. *The Reasoning Ability of Children of The Fourth, Fifth, and Sixth School Grades*. New York: Teachers College, Columbia University, 1910.

Brogden, Hubert E. "Statistical Theory and Design," *Annual Review of Psychology*, Vol 5 (1964), pp. 377-400.

Burt, Cyril. "The Development of Reasoning in School Children," *Journal of Experimental Pedagogy*, Vol. 5, No. 2 and 3 (1919), pp. 68-77, 121-127.

Cohen, Morris R. and Nagel, Ernest. *An Introduction to Logic and Scientific Method*. New York: Harcourt, Brace and Co., 1934.

Cronbach, Lee J. *Educational Psychology*. New York: Harcourt, Brace and Co., 1954.

Ennis, Robert H. "A Concept of Critical Thinking," *Harvard Educational Review*, Vol 32, No. 1 (1962), pp. 81-111.

________. "A Definition of Critical Thinking." In Richard E. Ripple (ed.), *Readings in Learning and Human Abilities: Educational Psychology*. New York: Harper and Row, 1964. Pp. 286-304. Also: *The Reading Teacher*, Vol. 17, No. 8 (May, 1964), pp. 599-612.

________. "Operational Definitions," *American Educational Research Journal*, Vol. 1, No. 3 (1964c), pp. 183-201.

Fairgrieve, M. MacCallum. "A Mass Test With Mr. Cyril Burt's Intelligence Questions," *Journal of Experimental Pedagogy*, Vol. 6 (1921), pp. 27-33.

Flavell, John H. *The Developmental Psychology of Jean Piaget*. D. Van Nostrand Co., Inc. Princeton, New Jersey: (1963).

Hill, Shirley Ann. "A Study of Logical Abilities in Children." Unpublished doctoral dissertation, Stanford University, 1961.

Inhelder, Barbel and Piaget, Jean. *The Growth of Logical Thinking from Childhood to Adolescence: An Essay on the Construction of Formal Operational Structures*. London: Routledge and Kegan Paul, 1958.

Miller, Elmer H. "A Study of Difficulty Levels of Selected Types of Fallacies in Reasoning and Their Relationships to the Factors of Sex, Grade Level, Mental Age and Scholastic Standing," *Journal of Educational Research*, Vol. 49, No. 2 (1955) pp. 123-129.

Moore, Dom Thomas Verner. "The Reasoning Ability of Children in the First Years of School Life," Studies in Psychology and Psychiatry from the Catholic University of America, Vol. 2, No. 2 (April, 1929).

Parsons, Charles. "Inhelder and Piaget's The Growth of Logical Thinking, II. A Logician's Viewpoint," British Journal of Psychology, Vol. 51, No. 1 (1960), pp. 75-84.

Piaget, Jean. Judgment and Reasoning in The Child. Paterson, New Jersey: Littlefield, Adams and Co., 1928.

________. The Psychology of Intelligence. London: Routledge and Kegan Paul, 1950.

________. Logic and Psychology. New York: Basic Books, Inc., 1953.

Ryan, Thomas A. "Multiple Comparisons in Psychological Research," Psychological Bulletin, Vol. 56, No. 1 (1954).

Smith, B. Othanel. "The Improvement of Critical Thinking," Progressive Education, Vol. 30, No. 5 (March, 1953), pp. 129-134.

Strawson, P. F. Introduction to Logical Theory. London: Methuen & Co., Ltd. 1952.

Sweeney, E. J. "Sex Difference in Problems Solving." Unpublished doctoral dissertation, Stanford University, 1953.

Warner, W. Lloyd. Social Class in America. New York: Harper and Brothers, 1949.

Wilkins, Minna Cheves. The Effect of Changed Material on Ability to Do Formal Syllogistic Reasoning. Archives of Psychology, No. 102, 1928.

Winch, W. H. "Children's Reasonings: Experimental Studies of Reasoning In School Children," The Journal of Experimental Pedagogy, Vol. 6 (1921), pp. 121-141, 199-212, 275-287.

Woodcock, Louise P. Life and Ways of the Two-Year-Old. New York: E. P. Dutton and Co., Inc., 1941.

Wright, Georg Henrik Von. Logical Studies. London: Routledge and Kegan Paul, Ltd., 1957.

GENERAL REFERENCES

Note: The following list of references includes those in the previous list of cited references.

Anderson, Howard C., Marcham, Frederick G., and Dunn, Seymour B. "An Experiment in Teaching Certain Skills of Critical Thinking," *Journal of Educational Research*: Vol. 38, No. 4 (December, 1944), pp. 241-251.

Arieta, S. *Interpretation of Schizophrenia*. New York: Brunner, 1955.

Arnold, Dwight L. "Testing Ability to Use Data in the Fifth and Sixth Grades," *Educational Research Bulletin*, Vol. 17, No. 9 (December 7, 1938), pp. 255-259, 278.

Baldwin, G. Bernard. *Inductive Reasoning Test*. Minneapolis, Minnesota: Educational Test Bureau, 1946.

Black, Max. *Critical Thinking* (Second Edition). New York: Prentice-Hall, Inc. 1952.

Bonser, Frederick G. *The Reasoning Ability of Children of the Fourth, Fifth, and Sixth School Grades*. New York: Teachers College, Columbia University, 1910.

Brogden, Hubert E. "Statistical Theory and Design," *Annual Review of Psychology*, Vol. 5 (1954), pp. 377-400.

Bruner, Jerome S., Goodnow, Jacqueline J., and Austin, George A. *A Study of Thinking*. New York: John Wiley & Sons, Inc., 1965.

Burt, Cyril. "The Development of Reasoning in School Children," *Journal of Experimental Pedagogy*, Vol. 5, No. 2 & 3 (1919), pp. 68-77, 121-127.

Chapman, L. J., and Chapman, J. P. "Atmosphere Effect Re-examined," *Journal of Experimental Psychology*, Vol. 58 (1959) pp. 220-226.

Chase, Stuart. *The Tyranny of Words*. New York: Harcourt, Brace and Company, 1938.

Cohen, Morris R. and Nagel, Ernest. *An Introduction to Logic and Scientific Method*. New York: Harcourt, Brace and Company, 1934.

Conant, James Bryant. *On Understanding Science*. New York: Mentor Books, 1951.

Cooperative Study of Evaluation in General Education of the American Council on Education. *A Test of Critical Thinking*, (Form G). New York: American Council on Education, 1952.

Crawshay-Williams, Rupert. Methods and Criteria of Reasoning. London: Routledge and Kegan Paul, 1957.

Cronbach, Lee J. Educational Psychology. New York: Harcourt, Brace and Company, 1954, p. 272.

Dennis, Wayne. "Piaget's Questions Applied to a Child of Known Environment," The Journal of Genetic Psychology, Vol. 61 (1942), pp. 307-320.

Deutsche, Jean Marquis. The Development of Children's Concepts of Causal Relations. Minneapolis: The University of Minnesota Press, 1937.

Dewey, John. How We Think. Boston: D. C. Heath & Co., 1933.

Dickinson, A. E., and Tyler, F. T. "An Experimental Study of the Generalizing Ability of Grade II Children," Journal of Educational Psychology, Vol. 35 (1944), pp. 432-441.

Dressel, Paul L., and Mayhew, Lewis B. General Education Explorations in Evaluation. Washington, D. C.: American Council on Education, 1954.

Educational Policies Commission. The Central Purpose of American Education. Washington, D. C.: National Educational Association of the United States, 1961.

Educational Testing Service, Cooperative Test Division. Logical Reasoning Test, Form B. Princeton, New Jersey: Educational Testing Service, 1950.

Ennis, Robert H. "Critical Thinking: More on Its Motivation," Progressive Education, May, 1956, pp. 75-78.

_______. "An Appraisal of the Watson-Glaser Critical Thinking Appraisal," Journal of Educational Research, Vol. 52 (1958), pp. 155-158.

_______. "Assumption Finding," In B. Othanel Smith and Robert H. Ennis (eds.), Language & Concepts in Education. Chicago: Rand McNally, 1961, pp. 161-178.

_______, and Millman, Jason. "Cornell Critical Thinking Test, Form X." Ithaca, New York: 1961a (mimeographed edition).

_______, and Millman, Jason. "Cornell Critical Thinking Test, Form Z." Ithaca, N. Y.: 1961b (mimeographed edition).

_______. "A Concept of Critical Thinking," Harvard Educational Review, Vol. 32, No. 1 (1962), pp. 81-111.

Ennis, Robert H. "A Definition of Critical Thinking," In Richard E. Ripple (ed.), Readings in Learning and Human Abilities: Educational Psychology. New York: Harper and Row, 1964. Pp. 286-304. Also: The Reading Teacher, Vol. 17, No. 8 (May, 1964), pp. 599-612.

_______. "Needed Research in Critical Thinking," A paper presented to the New England Regional Conference on Educational Research, Rhode Island State College, Providence, Rhode Island, November, 1964b. Mimeographed.

_______. "Operational Definitions," American Educational Research Journal, Vol. 1, No. 3 (1964c), pp. 183-201.

Fairgrieve, M. MacCallum. "A Mass Test with Mr. Cyril Burt's Intelligence Questions," Journal of Experimental Pedagogy, Vol. 6 (1921), pp. 27-33.

Flavell, John H. The Developmental Psychology of Jean Piaget. Princeton, New Jersey: D. Van Nostrand Co., Inc., 1963.

Glaser, Edward M. An Experiment in the Development of Critical Thinking. Teachers College Contributions to Education, Number 843. New York: Bureau of Publications, Teachers College, Columbia University, 1941.

Gordon, Raymond L. "The Effect of Attitude Toward Russia on Logical Reasoning," Journal of Social Psychology, Vol. 37 (1953), pp. 103-111.

Gottschalk, Louis. Understanding History. New York: Alfred A. Knopf, 1951.

Guillet, Cephas. "The Growth of a Child's Concepts," Pedagogical Seminary, Vol. 24 (1917), pp. 81-96.

Hazlitt, V. "Children's Thinking," British Journal of Psychology, Vol. 20 (1930), pp. 354-361.

Hempel, Carl G. Fundamentals of Concept Formation in Empirical Science. Chicago: University of Chicago Press, 1952.

Hendrix, G. "Developing a Logical Concept in Elementary Mathematics," Mathematics Teacher, Vol. 48 (1955), pp. 542-547.

Henle, M. "On Error in Deductive Reasoning," Psychological Report, Vol. 7 (1960), p. 80.

Henle, Mary. "On the Relation Between Logic and Thinking," Psychological Review, Vol. 4 (1962), pp. 366-378.

Henle, M., and Michael, M. "The Influence of Attitudes on Syllogistic Reasoning," Journal of Social Psychology, Vol. 44 (1956), pp. 115-127.

Hill, Shirley Ann. A Study of Logical Abilities in Children. Unpublished doctoral thesis, Stanford University, 1961.

Hood, Fred C. Developing the Ability to Assess the Results of Thinking. Urbana, Illinois: University of Illinois Bulletin, 1949.

Hospers, John. "What is Explanation?" In Antony Flew (ed.) Essays In Conceptual Analysis. London: Macmillan, 1956. Pp. 94-119.

Hyram, George H. "An Experiment In Developing Critical Thinking In Children," Journal of Experimental Education, Vol. 26 (December, 1957), pp. 125-132.

Inhelder, Barbel, and Piaget, Jean. The Growth of Logical Thinking from Childhood to Adolescence: An Essay on the Construction of Formal Operational Structures. London: Routledge & Kegan Paul, 1958.

Interpretation of Literature Tests. Princeton, New Jersey: The Cooperative Test Division of the Educational Testing Service, 1950.

The Iowa Tests of Educational Development. Tests 5, 6, 7, Form Y-3S, Science Research Associates, Chicago, Illinois, 1952.

Janis, I. L., and Frick, F. "The Relationship Between Attitudes Toward Conclusions and Errors in Judging Logical Validity of Syllogisms," Journal of Experimental Psychology, Vol.33 (1943), pp. 73-77.

Kane, R. B. "Some Effects of Beliefs About Conclusions of Arguments on the Ability to Judge the Validity of the Argument," Unpublished dissertation, University of Illinois, 1960.

Lee, Dorris May, and Bingham, Alma. "Intellectual Processes," Review of Educational Research, Vol. 29 (April, 1959), pp. 185-196.

Leeper, Robert. "Cognitive Processes," In S. S. Stevens (ed.), Handbook of Experimental Psychology. New York: John Wiley & Sons, Inc., 1951. Pp. 730-757.

Lefford, A. "The Influence of Emotional Subject Matter on Logical Reasoning," Journal of General Psychology, Vol. 34 (1946), pp. 127-151.

Logical Reasoning Test, Form B. Princeton, New Jersey: The Cooperative Test Division of the Educational Testing Service, 1950.

Luchins, A. S. "On the Awareness and Denotation of Contradictions," Journal of General Psychology, Vol. 71 (1964), pp. 233-246.

Macmillan, C. J. B. and Ennis, Robert H. "Reviews of Works Related to Critical Thinking," Critical Thinking Research Series, Junior High School Project, First Ed. Ithaca, New York: 1961, (mimeographed) (out of stock).

Maier, Norman R. F. "Reasoning in Children," The Journal of Comparative Psychology, Vol. 21 (June, 1936), pp. 357-366.

Marcham, Frederick George. "Teaching Critical Thinking and the Use of Evidence," The Quarterly Journal of Speech, Vol. 31, No. 3 (October, 1945), pp. 362-368.

McCarthy, Philip J. "Sampling," Bulletin No. 15 of the New York State School of Industrial and Labor Relations, Ithaca, New York: 1951.

Michigan State College Board of Examiners. A Test of Problem Solving. Junior and Senior High School Edition, Form B. East Lansing: Michigan State College, 1954a.

Michigan State College Board of Examiners. A Test of Problem Solving. High School Edition, Form A. East Lansing: Michigan State College, 1954b.

Miller, Elmer H. "A Study of Difficulty Levels of Selected Types of Fallacies in Reasoning and Their Relationships to the Factors of Sex, Grade Level, Mental Age and Scholastic Standing," Journal of Educational Research, Vol. 49, No. 2 (1955), pp. 123-129.

Mohandessi, K. "Development of a Test to Measure Logical Reasoning of High School Students," (unpublished master's thesis) University of Illinois, 1956.

Moore, Dom Thomas Verner. "The Reasoning Ability of Children in The First Years of School Life," Studies in Psychology and Psychiatry, Edward A. Pace (ed.), Vol. 2, No. 2 (1929).

Morgan, J. J. B., and Carrington, D. H. "Graphic Instruction in Relational Reasoning," Journal of Educational Psychology, Vol. 35 (1944), pp. 536-544.

________, and Morton, J. T. "The Distortion of Syllogistic Reasoning Produced by Personal Conviction," Journal of Social Psychology, Vol. 20 (1944), pp. 39-59.

Navarra, John Gabriel. The Development of Scientific Concepts In a Young Child. New York: Bureau of Publications, Teachers College, Columbia University, 1955.

Oakes, Mervin E. Children's Explanations of Natural Phenomena. New York: Bureau of Publications, Teachers College, Columbia University, 1947.

Oliver, Donald W., and Shaver, James P. "The Analysis of Public Controversy: A Study in Citizenship Education," Vol. 1 and 2. A Report from the Laboratory for Research in Instruction, Graduate School of Education, Harvard University, 1962.

Parsons, Charles. "Inhelder and Piaget's The Growth of Logical Thinking," II A Logician's Viewpoint. British Journal of Psychology, 1960, pp. 75-84.

Pearson, William A., and Hyram, George H. Pearson-Hyram Test of Critical Thinking. St. Louis, Missouri: Saint Louis University, 1958.

Peterson, George M. "An Empirical Study of The Ability to Generalize," Journal of General Psychology, Vol. 6 (1932), pp. 90-113.

Piaget, J. The Language and Thought of the Child. New York: Harcourt, Brace and Co., 1925.

_______. Judgment and Reasoning in the Child. Paterson, N. J.: Littlefield, Adams and Company, 1959 (first printed, 1928).

_______. The Psychology of Intelligence. London: Routledge and Kegan Paul limited, 1950.

_______. Logic and Psychology. New York: Basic Books, Inc., 1957.

Richter, Maurice N. Jr. "The Theoretical Interpretation of errors in Syllogistic Reasoning," Journal of Psychology, Vol. 43 (1958), pp. 341-344.

Robinson, Richard. Definition. Oxford: Oxford University Press, 1954.

Russell, David H. Children's Thinking. Boston: Ginn and Co., 1956.

Ryan, Thomas A. "Multiple Comparisons in Psychological Research," Psychological Bulletin, Vol. 56, No. 1 (January 1959), pp. 26-47.

Scheffler, Israel. "Explanation, Prediction, and Abstract Entities," British Journal for the Philosophy of Science, Vol. 7 (1956-7), pp. 293-309.

Sell, DeWitt E. Evaluation Aptitude Test. Chicago, Illinois: Psychometric Affiliates, 1952.

Sells, S. B. "The Atmosphere Effect," Archives of Psychology, 1936, No. 200.

Shelley, H. P., and Davis, R. E. "Relationship of Attitude to Logical Problem-Solving," Psychology Report, Vol. 3 (1957), pp. 525-530.

Shenfeld, N. "Tolerant and Intolerant Attitudes and Logical Thinking," Dissertation Abstracts, Vol. 18 (1958), p. 2240.

Smith, B. Othanel. "The Improvement of Critical Thinking," Progressive Education, Vol. 30, No. 5 (March, 1953), pp. 129-134.

________. "Logic, Thinking, and Teaching," Educational Theory, Vol. 7, No. 4 (October, 1957), pp. 225-233.

________. "Critical Thinking," Recent Research and Developments and Their Implications for Teacher Education. 13th Yearbook of the American Association of Colleges for Teacher Education, Washington, D. C. (1960).

Stewart, Daniel K. "The Laws of Thought: An Hypothesis," Psychology Report, Vol. 7 (1960), pp. 229-232.

Strawson, P. F. Introduction to Logical Theory. London: Methuen & Co., Ltd., 1952.

Suppes, Patrick and Binford, Frederick. "Experimental Teaching of Mathematical Logic in the Elementary School," The Arithmetic Teacher, March, 1965, pp. 187-195.

Sweeney, E. J. "Sex Difference in Problems Solving." Unpublished doctoral dissertation, Stanford University, 1953.

Taba, Hilda. Thinking in Elementary School Children. (Unpublished summary of a Cooperative Research Study). San Francisco, California: 1964.

Test of Application of Principles in Biology, Test of Application of Principles in General Science, and Test of Application of Principles in Physical Science. (Form A), Princeton, New Jersey: Cooperative Test Division of the Educational Testing Service, 1950.

Thistlewaite, D. L. "Attitude and Structure as Factors in the Distortion of Reasoning," Journal of Abnormal Social Psychology, Vol. 45 (1950), pp. 442-458.

Thompson, Jane. "The Ability of Children of Different Grade Levels to Generalize on Sorting Tests," The Journal of Psychology, Vol. 2 (1941), pp. 119-126.

Thorndike, E. L. "The Effect of Changed Data Upon Reasoning," Journal of Experimental Psychology, Vol. 5 (1922), pp. 33-38.

Thouless, R. H. "Effect of Prejudice on Reasoning," British Journal of Psychology, Vol. 50 (1959), pp. 289-293.

Toulmin, Stephen Edelston. The Uses of Argument. Cambridge: Cambridge University Press, 1958.

Valentine, Charles W. Intelligence Tests for Children (Sixth Edition). London: Methuen and Co., Ltd., 1958 (First edition, 1945).

Von Domarus, E. "The Specific Laws of Logic in Schizophrenia," In J. S. Kasmin, (ed.), Language & Thought in Schizophrenia. Berkeley: University of California Press, 1944.

Warner, W. Lloyd. Social Class in America. New York: Harper and Brothers, 1949.

Wason, P. C. "The Effect of Self Contradiction on Fallacious Reasoning," Quarterly Journal of Experimental Psychology, Vol. 16, No. 1 (1964), pp. 30-34.

Watson, Goodwin, and Glaser, Edward Maynard. Watson-Glaser Critical Thinking Appraisal, Forms Am and Bm. Yonkers-on-Hudson: World Book Co., 1952.

Welch, Livingstone. "Some Aspects of the Development of Inductive Reasoning," New York Academy of Science, Vol. 4 (1942), pp. 218-221.

________. "Methods Used by Children in Solving Inductive Reasoning Problems," Journal of Psychology, Vol. 14 (1943), pp. 269-275.

________, and Long, Louis. "Comparison of the Reasoning Ability of Two Age Groups," The Journal of Genetic Psychology, Vol. 62 (1943), pp. 63-76.

Wigmore, John Henry. A Students' Textbook of the Laws of Evidence. Brooklyn: The Foundation Press, 1935.

Wilkins, Minna Cheves. The Effect of Changed Material on Ability to Do Formal Syllogistic Reasoning. Archives of Psychology, No. 102 (1928).

Williams, E. B. "Deductive Reasoning in Schizophrenia," Journal of Abnormal Social Psychology, Vol. 69, No. 1 (1964), pp. 47-61.

Willis, Mary, and Domoncovich, H. A. Cooperative Literary Comprehension and Appreciation Test (Form T). Princeton, New Jersey: The Cooperative Test Division of the Educational Testing Service, 1943.

Winch, W. H. "Children's Reasonings: Experimental Studies of Reasoning In School Children," The Journal of Experimental Pedagogy, Vol. 6 (1921), pp. 121-141, 199-212, 275-287.

Woodcock, Louise P. Life and Ways of the Two-Year-Old. New York: E. P. Dutton & Co., Inc., 1941.

Woodworth, R. S., and Sells, P. B. "An Atmosphere Effect in Formal Syllogistic Reasoning," *Journal of Experimental Psychology*, Vol. 18 (1935), pp. 451-460.

Wright, Georg Henrik Von. *Logical Studies*. The Humanities Press Inc., London: Routledge & Kegan Paul Ltd., 1957.

APPENDIX A. TABLES AND GRAPHS.

The tables and graphs presented in this part of the appendix are listed at the beginning of the report under the heading, "Tables and Graphs".

TABLE A-1. Individual Item and Item Group Mean Difficulty Indices by Grades on the Conditional Reasoning Test for All Subjects on the Pre-test and Also for LDT Pre-and Post-tests.

	All* Sub-jects Pre	LDT Pre	LDT Post	All* Sub-jects Pre	LDT Pre	LDT Post	All* Sub-jects Pre	LDT Pre	LDT Post	All* Sub-jects Pre	LDT Pre	LDT Post
Grades		05			07			09			11	
N =	102	27	27	99	24	24	80	17	17	78	26	26
Item Group 01.** If p then q. p. ***: q.												
Item Number												
07	68	63	78	61	58	96	78	53	94	76	81	96
14	78	74	63	76	67	71	75	71	47	76	92	100
19	86	85	96	95	100	96	90	94	94	91	100	100
27	74	63	70	75	88	83	69	59	83	81	81	92
31	48	59	78	72	79	92	75	88	82	73	81	96
40	75	67	78	71	62	63	78	82	65	77	92	96
Mean	71	68	77	75	76	83	77	74	78	79	88	97
Item Group 02. If p then q. Not p. : Not q.												
Item Number												
09	34	41	19	41	33	33	43	29	47	49	39	85
13	24	37	37	34	29	50	34	35	18	49	54	85
18	32	37	44	23	21	25	23	06	24	39	31	73
23	05	00	04	13	04	13	09	00	00	12	19	85
26	27	44	33	42	25	33	33	12	29	40	39	85
34	15	22	11	10	13	13	09	06	24	24	19	89
Mean k	23	30	25	28	21	28	25	15	24	35	34	84

* LDT's & LNDT-1's & LNDT-2's. ** The basic symbolic form for each item group is presented. Variations are given in Chapter Iv. *** The symbol ":" is used to introduce the proposed statement.

TABLE A-1 cont.

Grades		05			07			09			11	
N =	102	27	27	99	24	24	80	17	17	78	26	26
Item Group 03. If p then q. q : p.												
Item Number												
11	21	33	33	30	29	46	29	24	29	31	35	96
24	24	33	44	32	25	42	35	35	24	44	46	100
30	12	11	19	21	21	21	24	35	47	33	39	89
32	28	41	63	52	42	50	51	47	59	67	77	100
37	16	15	26	14	21	33	30	29	18	09	42	100
41	06	11	26	11	08	17	20	06	06	31	42	100
Mean	18	24	35	27	24	35	32	29	31	36	41	94
Item Group 04. If p then q. Not q. : Not p.												
Item Number												
08	68	63	52	82	88	79	78	76	71	82	89	81
16	37	30	44	52	46	58	34	24	65	51	65	77
22	62	70	59	73	54	79	59	53	53	56	50	85
29	65	56	70	84	92	79	81	82	71	89	92	92
35	51	52	22	62	67	67	53	41	47	53	69	65
39	52	37	44	61	62	79	54	76	65	59	62	89
Mean	56	51	49	69	68	74	60	59	63	65	71	82
Item Group 05. If p then q. If q then r. : If p then r.												
Item Number												
45	46	48	59	49	54	71	44	23	35	63	77	89
49	45	37	52	61	63	88	61	65	65	77	89	96
52	60	67	67	70	71	75	73	88	65	78	89	77
55	79	81	89	91	88	88	88	88	94	92	96	100
66	66	59	74	71	75	83	68	71	59	67	85	92
73	44	44	63	68	88	75	65	71	71	80	89	89
Mean	57	56	67	68	73	80	66	68	65	76	87	91

TABLE A-1 cont.

Grades		05			07			09			11	
N =	102	27	27	99	24	24	80	17	17	78	26	26
Item Group 06. If p then q. : If not q then not p.												
Item Number												
46	59	44	67	68	71	67	55	59	35	73	85	77
50	54	44	74	69	83	75	63	76	59	72	73	92
56	56	44	52	68	54	63	61	47	35	51	50	89
61	58	56	74	61	71	79	55	35	41	54	54	85
69	61	44	67	71	75	50	61	71	29	59	69	77
74	52	33	56	62	75	67	51	59	24	53	69	96
Mean	57	44	65	66	71	67	58	58	37	65	67	86
Item Group 07. If p then q. : If q then p.												
Item Number												
44	14	15	26	31	21	13	28	35	41	49	62	73
57	19	15	26	22	08	25	33	12	24	42	42	81
59	11	11	19	30	21	13	29	35	41	39	50	85
64	16	22	19	19	04	17	21	29	18	37	42	81
70	11	07	19	16	17	21	24	12	24	41	54	77
77	15	19	15	25	21	29	38	47	47	51	65	81
Mean	14	15	21	24	15	20	29	28	33	43	53	80
Item Group 08. p only if q. Not q. : Not p.												
Item Number												
12	78	78	74	85	92	88	83	71	76	89	100	85
15	72	70	70	88	83	79	86	88	82	94	96	77
21	84	63	67	80	83	63	85	76	82	85	96	81
25	46	37	37	67	67	67	54	41	65	69	85	65
36	63	52	70	82	100	83	80	88	94	91	100	89
42	69	63	59	83	75	79	80	65	76	87	85	81
Mean	69	71	63	81	83	77	78	71	79	86	88	80

TABLE A-4 cont.

Grades		05			07			09			11	
N =	102	27	27	99	24	24	80	17	17	78	26	26
Item Group 09. p only if q. p. : q.												
Item Number												
10	77	63	85	79	83	92	90	88	94	95	100	85
17	87	89	93	86	88	96	91	100	94	94	100	81
20	79	81	67	89	88	88	83	88	94	96	96	85
28	64	67	63	85	96	92	83	82	83	95	96	77
33	36	33	37	48	54	79	38	41	53	53	60	77
38	81	89	89	84	88	100	88	82	82	94	96	73
Mean	71	70	72	78	83	91	79	80	84	88	93	80
Item Group 10. Comb. 1. And 1 or 5.												
Item Number												
43	50	55	33	71	79	75	65	59	65	81	89	73
51	49	48	41	75	71	83	74	76	76	85	96	89
62	72	71	59	85	92	96	81	76	64	87	92	100
67	54	41	48	70	67	83	69	71	65	81	96	96
72	45	33	52	55	67	58	53	47	47	55	65	89
76	59	52	56	73	79	75	71	82	53	77	92	96
Mean	55	50	48	71	76	78	69	68	62	78	88	91
Item Group 11. p if and only if q. p or not p. : q or not q.												
Item Number												
47	64	52	44	71	79	46	76	70	59	73	73	73
54	66	52	48	73	79	79	71	65	41	68	73	62
58	20	22	22	33	50	50	31	18	29	54	81	58
60	62	56	63	78	79	83	80	94	65	85	92	69
63	65	55	67	74	71	54	65	65	41	59	58	89
78	52	44	44	68	75	75	61	59	59	59	54	77
Mean	55	47	48	66	72	65	64	62	49	66	72	71

TABLE A.4 cont.

Grades		05			07			09			11	
N =	102	27	27	99	24	24	80	17	17	78	26	26
Item Group 12. Comb. p only if q. q. : p and l.												
Item Number												
48	30	33	26	20	21	21	30	29	18	35	42	69
53	20	26	41	23	17	13	24	24	23	37	42	62
65	29	44	33	10	13	08	10	36	00	13	08	50
68	25	22	30	24	21	21	20	18	12	27	08	58
71	29	37	30	16	17	00	13	24	06	09	04	69
75	25	26	22	13	08	13	20	35	00	09	04	65
Mean	26	31	30	18	16	13	19	28	10	22	18	62

TABLE A-2. Individual Item and Item Group Mean Difficulty Indices by Grades on the Class Reasoning Test for All Subjects on the Pre-test and Also for LDT Pre- and Post-tests.

	All* Sub-jects Pre	LDT Pre	LDT Post	All* Sub-jects Pre	LDT Pre	LDT Post	All* Sub-jects Pre	LDT Pre	LDT Post	All* Sub-jects Pre	LDT Pre	LDT Post	All* Sub-jects Pre	LDT Pre	LDT Post
Grades		04			06			08			10			12	
N =	94	25	25	103	25	25	100	27	27	75	22	22	72	24	24
Item Group 01.** All As are Bs. : *** At least some As are not Bs.															
Item Number															
08	79	76	60	91	96	100	95	100	93	96	100	100	93	100	100
16	80	72	68	94	96	96	94	96	96	91	96	96	100	100	100
22	69	80	56	85	92	92	88	96	85	92	91	86	92	96	96
29	85	80	68	99	100	100	92	93	100	95	91	96	97	100	100
35	76	80	64	91	92	100	95	100	93	95	100	96	94	100	92
39	68	68	68	84	88	96	85	89	89	93	96	100	96	100	100
Mean	76	76	64	91	91	97	92	95	93	93	96	96	95	99	98
Item Group 02. All As are Bs. All Bs are Cs. : All As are Cs.															
Item Number															
07	76	64	76	85	96	92	82	86	89	94	91	86	92	96	96
14	68	76	80	80	96	80	80	86	96	95	91	96	92	96	100
19	26	36	72	35	28	84	55	86	93	71	68	82	78	68	96
27	92	92	80	93	100	100	94	100	96	99	96	96	97	100	100
31	49	60	56	62	64	88	65	86	89	83	82	86	64	76	100
40	25	80	56	35	60	72	30	75	78	55	77	91	74	83	92
Mean	56	68	70	65	74	86	68	72	90	82	84	90	83	89	98

---

* LDT's & LNDT-1's & LNDT-2's. ** The basic symbolic form for each item group is presented.

*** Variations are given in Chapter IV.

TABLE A-2 cont.

Grades		04			06			08			10			12	
N =	94	25	25	103	25	25	100	27	27	75	22	22	72	24	24
Item Group 03. All As are Bs. : All Bs are As.															
Item Number															
11	25	20	20	33	36	60	40	39	78	64	64	73	76	92	100
24	30	52	64	53	60	60	65	76	78	77	86	91	85	96	92
30	21	12	08	28	36	32	31	21	41	39	41	64	44	54	79
32	40	40	52	60	60	56	67	61	70	88	96	96	88	92	96
37	40	56	72	48	44	72	62	71	85	76	82	96	83	96	96
41	25	24	12	35	36	56	30	39	78	55	59	82	74	83	88
Mean	30	34	38	43	45	56	49	51	72	67	71	84	75	85	92
Item Group 04. All As are Bs. All Cs are Bs. : At least some Cs are As.															
Item Number															
09	57	72	28	50	48	60	58	50	41	59	73	73	72	71	96
13	50	56	52	47	48	56	51	39	56	67	59	77	75	100	92
18	53	60	40	50	40	64	58	64	70	72	69	91	78	96	88
23	20	32	24	20	28	32	34	50	19	36	41	64	44	63	83
26	48	56	56	62	60	60	62	57	81	88	100	82	89	92	88
34	32	20	24	27	20	56	23	32	37	27	23	82	42	38	83
Mean	43	49	37	43	41	55	48	49	51	58	61	80	68	77	78
Item Group 05. No As are Bs. : No Bs are As.															
Item Number															
10	70	64	68	73	72	84	75	82	89	91	82	82	83	96	96
17	77	80	96	93	96	92	93	96	96	88	91	100	92	100	96
20	95	88	80	97	96	88	98	100	100	99	96	100	97	100	100
28	50	68	68	67	68	76	77	86	93	87	100	86	81	88	96
33	30	12	32	32	24	64	30	25	52	47	55	59	56	71	88
38	67	80	68	62	56	72	25	78	93	69	59	77	71	75	96
Mean	65	65	69	71	69	80	66	78	87	80	80	84	80	88	95

TABLE A-2 cont.

Grades		04			06			08			10			12	
N =	94	25	25	103	25	25	100	27	27	75	22	22	72	24	24
Item Group 06. All As are Bs. No Cs are Bs. : At least some As are Cs.															
Item Number															
12	51	36	56	80	92	84	78	86	96	91	86	82	88	88	92
15	61	60	80	64	68	60	67	86	70	60	50	59	79	88	100
21	56	48	48	79	92	92	82	93	100	91	86	86	89	96	100
25	70	64	72	87	92	100	88	96	85	91	96	91	92	96	92
36	53	60	52	63	68	80	76	100	85	89	91	91	81	88	83
42	79	56	56	88	80	84	83	89	93	97	77	100	94	88	92
Mean	62	54	61	77	82	84	79	92	88	87	81	85	87	91	93
Item Group 07. All As are Bs. No Cs are As. : No Cs are Bs.															
Item Number															
44	32	36	28	25	20	52	39	46	67	63	77	77	68	79	88
57	33	32	52	45	56	48	39	46	52	52	36	73	63	67	63
59	34	36	08	35	40	32	22	11	37	31	41	68	36	42	75
64	22	20	12	20	24	32	32	32	48	48	27	82	60	58	67
70	48	52	48	50	56	60	30	46	67	80	82	77	79	88	83
77	34	40	56	53	52	56	57	61	81	79	73	77	72	88	88
Mean	34	36	34	38	41	47	37	40	59	59	56	75	63	70	77
Item Group 08. No As are Bs. No Cs are Bs. : At least some As are Cs.															
Item Number															
48	38	48	40	30	32	36	42	36	52	56	55	73	65	75	88
53	40	48	24	52	44	64	57	68	81	77	86	77	79	100	92
65	38	32	36	49	40	64	59	54	63	64	55	91	78	71	92
68	46	44	40	44	48	52	66	68	89	76	68	86	79	92	96
71	31	24	36	41	44	56	44	46	67	73	73	82	74	92	83
75	29	40	28	40	44	28	42	50	52	59	50	96	54	67	88
Mean	37	39	34	42	42	50	52	54	67	68	64	84	72	83	90

TABLE A-2 cont.

Grades	04			06			08			10			12		
N =	94	25	25	103	25	25	100	27	27	75	22	22	72	24	24
Item Group 09. All As are Bs. : All Non-Bs are also Non-As.															
Item Number															
45	33	24	52	49	56	64	48	61	52	61	59	77	68	67	88
49	55	72	44	47	40	56	54	68	67	57	64	68	65	83	79
52	54	52	56	56	52	64	63	64	70	76	82	73	76	86	83
55	52	36	52	56	52	64	58	71	59	60	50	64	6	67	88
66	63	64	64	83	84	96	81	100	81	93	96	91	83	96	96
73	49	48	52	48	40	68	47	54	59	52	55	64	50	71	83
Mean	51	49	53	56	54	69	59	70	65	68	68	73	58	78	86
Item Group 10. All As are Bs. All Non-Cs are Non-Bs. : At least some As are not Cs.															
Item Number															
43	28	20	48	34	32	24	39	46	44	45	50	55	51	50	63
51	37	56	44	41	68	60	59	61	63	76	77	64	72	96	79
62	69	56	44	87	76	92	79	78	74	87	73	73	75	50	88
67	34	44	32	35	48	28	32	43	37	36	18	41	28	50	63
72	45	40	20	47	56	60	41	54	48	43	55	50	42	54	67
76	49	64	44	46	56	56	65	86	63	69	82	55	64	79	75
Mean	44	47	39	48	56	53	53	61	55	59	59	56	55	63	73
Item Group 11. All As are Bs. All Cs are Ds. All Bs are Cs. : At least some As are not Ds.															
Item Number															
47	61	52	44	63	76	68	70	78	85	81	77	64	72	67	83
54	59	60	68	75	68	84	75	75	85	93	96	100	92	100	88
58	56	68	52	74	80	80	62	86	85	71	73	64	79	88	75
60	30	16	24	48	56	64	66	82	74	80	82	68	79	88	88
63	75	44	40	79	40	68	80	86	85	93	77	91	89	96	92
78	33	36	28	36	36	24	45	79	81	57	41	77	58	92	83
Mean	52	46	43	62	59	65	66	81	83	79	74	78	78	88	85

TABLE A-4 cont.

Grades		04			06			08			10			12	
N =	94	25	25	103	25	25	100	27	27	75	22	22	72	24	24
Item Group 12. No Bs are Cs. No Ds are Cs. All As are Bs. : No Ds are As.															
Item Number															
46	63	56	68	63	64	84	71	89	85	85	82	91	85	79	100
50	37	32	48	34	32	48	55	68	78	57	55	77	59	71	92
56*															
61	47	52	60	36	12	64	56	78	70	57	59	77	69	71	100
69	54	48	56	67	56	72	64	75	78	77	77	77	68	88	92
74	49	44	56	63	52	80	73	74	85	84	86	91	78	96	88
Mean	50	46	58	53	43	70	78	77	79	72	72	83	72	81	94

* Item 56 turned out to be a defective item.

Table A-3. Individual Item and Item Group Mean Discrimination Indices By Grades on the Conditional Reasoning Test for All Subjects on the Pre-Test.

Grades N*=	05 102	07 99	09 80	11 78
Item Group 01.				
Item Number				
07	26	19	23	43
14	11	30	0	43
19	30	15	14	10
27	59	22	27	29
31	56	52	27	52
40	41	56	23	33
Mean	37	32	19	35
Item Group 02.				
Item Number				
09	-26	11	5	43
13	-15	26	9	29
18	-15	4	-18	5
23	0	15	9	33
26	-22	22	23	48
34	-30	22	5	10
Mean	-18	17	5	28
Item Group 03.				
Item Number				
11	- 7	7	5	38
24	-30	-22	5	29
30	-30	7	- 9	38
32	4	26	0	43
37	- 7	15	5	-14
41	- 7	4	18	43
Mean	-13	6	4	29
Item Group 04.				
Item Number				
08	41	15	27	24
16	11	56	14	67
22	59	19	36	5
29	70	37	41	38
35	48	41	5	-14
39	70	37	22	33
Mean	50	34	24	25

Table A-3 cont.

Grades N =	05 102	07 99	09 80	11 78
Item Group 05.				
Item Number				
45	22	30	32	57
49	33	59	18	0
52	37	37	23	48
55	37	30	9	19
66	30	41	32	48
73	63	63	32	43
Mean	37	43	24	36
Item Group 06.				
Item Number				
46	41	41	14	38
50	41	30	14	14
56	37	22	32	0
61	67	37	0	-14
69	70	26	9	19
74	48	30	14	29
Mean	51	31	14	14
Item Group 07.				
Item Number				
44	- 4	4	9	43
57	0	22	9	48
59	-19	7	41	62
64	-22	11	32	43
70	- 7	4	27	52
77	-22	7	18	71
Mean	12	9	23	53
Item Group 08.				
Item Number				
12	41	33	27	33
15	59	33	23	14
21	33	41	23	5
25	41	33	36	76
36	63	33	27	29
42	48	37	55	29
Mean	48	35	32	31

Table A-3 cont.

Grades	05	07	09	11
N =	102	99	80	78
Item Group 09.				
Item Number				
10	30	52	27	19
17	37	26	14	19
20	48	33	36	10
28	74	11	32	10
33	- 4	52	0	67
38	52	26	23	14
Mean	40	33	22	23
Item Group 10.				
Item Number				
43	56	46	55	57
51	48	63	55	48
62	22	41	14	38
67	33	56	50	43
72	33	30	36	43
76	26	41	27	48
Mean	36	46	39	46
Item Group 11.				
Item Number				
47	48	7	5	5
54	52	22	36	10
58	19	26	36	76
60	56	44	32	29
63	74	41	27	14
78	63	41	36	5
Mean	52	30	29	23
Item Group 12.				
Item Number				
48	-15	-11	0	0
53	15	18	18	19
65	- 7	- 4	0	-14
68	0	0	5	-33
71	11	15	0	- 5
75	- 7	-22	- 5	0
Mean	0	1	3	6

---

* The top and bottom groups which each were compared were made up of 27% of this N.

Table A-4. Individual Item and Item Group Mean Discrimination Indices by Grades on Class Reasoning Test for All Subjects on the Pre-Test.

Grades	04	06	08	10	12
N*=	94	103	100	75	72
Item Group 01.					
Item Number					
08	44	21	15	0	10
16	40	18	7	10	0
22	52	32	30	5	15
29	36	0	26	5	5
35	68	14	19	5	15
39	80	50	30	5	15
Mean	53	23	21	4	10
Item Group 02.					
Item Number					
07	40	21	33	10	5
14	28	29	30	5	25
19	32	25	59	30	45
27	20	11	19	0	0
31	68	68	52	15	30
40	8	11	30	30	15
Mean	33	27	37	15	20
Item Group 03.					
Item Number					
11	36	32	48	50	60
24	28	43	37	30	40
30	0	25	- 4	20	45
32	52	43	52	20	25
37	24	61	52	35	45
41	4	39	52	80	55
Mean	24	41	40	39	45
Item Group 04.					
Item Number					
09	36	36	15	5	30
13	12	21	37	40	30
18	36	29	56	50	40
23	12	0	41	30	60
26	16	57	33	10	20
34	12	0	7	20	25
Mean	21	24	32	26	34

Table A-4 cont.

Grades N =	04 94	06 103	08 100	10 75	12 72
Item Group 05.					
Item Number					
10	36	18	30	5	25
17	52	14	4	20	15
20	16	7	4	5	10
28	32	50	33	-10	10
33	- 4	11	33	35	70
38	48	-14	15	10	5
Mean	30	14	20	11	23
Item Group 06.					
Item Number					
12	40	29	52	5	25
15	4	- 4	33	55	40
21	80	43	30	15	35
25	40	14	7	- 5	5
36	36	71	56	15	35
42	40	7	52	10	15
Mean	40	27	38	16	26
Item Group 07.					
Item Number					
44	16	25	52	40	55
57	-12	43	37	60	55
59	0	71	- 7	-15	- 5
64	-16	29	41	35	70
70	32	61	44	30	40
77	20	39	59	40	65
Mean	7	45	38	32	47
Item Group 08.					
Item Number					
48	16	32	26	45	50
53	24	54	59	15	45
65	32	43	33	60	20
68	44	54	44	40	50
71	36	43	52	20	50
75	16	25	44	50	60
Mean	28	42	43	38	46

Table A-4 cont.

Grades N =	04 94	06 103	08 100	10 75	12 72
Item Group 09.					
Item Number					
45	40	25	41	25	45
49	32	29	37	20	55
52	32	36	37	0	5
55	40	11	22	35	-10
66	68	39	44	5	40
73	37	-14	15	20	25
Mean	42	21	33	18	27
Item Group 10.					
Item Number					
43	24	43	22	10	20
51	36	50	52	45	45
62	24	29	44	0	0
67	16	14	26	5	0
72	28	7	30	20	10
76	52	46	63	45	50
Mean	30	32	40	21	21
Item Group 11.					
Item Number					
47	40	29	30	15	40
54	60	14	52	5	15
58	56	43	33	30	35
60	44	68	74	15	55
63	12	0	48	10	30
78	20	11	37	40	55
Mean	39	27	46	19	38
Item Group 12.					
Item Number					
46	20	50	37	50	25
50	32	36	78	40	70
56	0	- 7	19	5	0
61	20	14	33	45	40
69	12	25	41	0	65
74	56	43	48	25	50
Mean	23	27	43	28	42

* The top and bottom groups which each were compared were made up of 27% of this N.

TABLE A-5. Conditional Reasoning Adjusted-Mean Post-Test Comparisons of Students to Whom Logic Was Deliberately Taught and Students to Whom Logic Was Not Deliberately Taught.

Grade	5		7		9		11	
Group	LDT	LNDT-1	LDT	LNDT-1	LDT	LNDT-1	LDT	LNDT-1
N=	27	26	24	25	17	23	26	22
Total Score	46.4	45.6	55.4	56.2	47.3	54.3*	77.6*	62.4
Component								
CF	24.2	24.5	28.3	29.0	25.7	28.1*	38.0*	31.8
SY	6.2	5.9	7.4	7.5	6.3	7.0	10.0*	7.8
SU	5.6	5.5	6.5	6.2	5.2	6.0	9.2*	6.5
Item Group								
1	4.6	4.4	5.2	5.1	4.7	5.2	5.7	5.2
2	1.4	1.8	1.8	1.3	1.4	1.2	5.1	2.0
3	2.1*	1.0	2.1	1.2	1.7	1.5	4.8	1.7
4	3.1	3.4	4.4	4.6	3.9	3.9	4.8	3.9
5	4.0	3.9	4.7	4.8	3.8	4.6	5.2	5.1
6	4.0	3.9	4.0	4.1	2.3	3.8	5.1	3.4
7	1.2	1.3	1.2	1.8	1.8	2.4	4.7	2.6
8	3.9	4.0	5.2	4.6	5.1	4.4	4.6	5.3
9	4.0	4.5	5.5	5.1	5.0	5.0	4.7	5.7
10	3.0	3.7	4.7	4.4	4.7	3.9	5.1	5.5
11	2.9	3.2	3.7	4.6	3.1	4.2	4.2	4.4
12	1.8	1.2	1.0	1.1	0.6	1.1	3.8	1.2

* An asterisk is placed by each statistically significantly superior adjusted mean.

Note: These adjusted means are part of the outcome of an analysis of covariance in which IQ and pre-test score were held constant.

TABLE A-6. Class Reasoning Adjusted-Mean Post-Test Comparisons of Students to Whom Logic Was Deliberately Taught and Students to Whom Logic Was Not Deliberately Taught.

Grade	4		6		8		10		12	
Group	LDT	LNDT-1	LDT	LNDT-1	LDT	LNDT-1	LDT	LNDT-1	LDT	LNDT-1
N=										
Total Score	44.8	47.3	63.8*	55.3	68.1	65.4	81.4	77.3	87.8	82.6*
Component										
CF	26.2	26.8	34.8*	31.2	36.7	35.1	40.7	41.0	43.7	42.1
SY	5.1	4.9	6.7*	5.3	7.5*	6.1	9.1	7.9	10.3*	8.8
SU	4.7	5.3	7.0	6.2	7.8	7.3	9.6	9.1	10.5	10.0
Item Group										
1	3.9	4.3	5.7	5.3	5.6	5.7	5.7	6.0	5.9	5.9
2	4.1	4.0	4.9	4.6	5.3	4.8	5.4	5.5	5.8	5.8
3	2.2	2.4	3.4	2.7	4.2	3.8	5.2	4.6	5.5	5.3
4	2.1	2.7	3.5	2.5	3.1	2.8	4.8	3.9	5.3	4.8
5	3.9	4.1	4.8	4.4	5.2	4.6	5.1	5.2	5.7	5.2
6	3.8	3.2	4.7	4.4	5.2	4.5	5.2	5.5	5.6	5.4
7	2.0	2.3	2.9	2.3	3.5	2.9	4.4	4.1	4.7	4.2
8	2.1	2.2	3.1	2.8	4.1	2.8	5.2	4.4	5.5	4.9
9	3.2	2.9	4.0	3.4	3.7	4.0	4.5	4.0	5.2	4.8
10	2.3	2.2	3.6	2.7	3.2	3.4	3.5	4.5	4.2	4.3
11	2.6	2.6	3.8	3.4	4.7	4.6	4.9	5.4	5.1	5.2
12	4.0	3.8	4.6	3.7	4.9	4.1	5.2	5.5	5.7	5.4

* An asterisk is placed by each statistically significantly superior adjusted mean.

Note: These adjusted means are part of the outcome of an analysis of covariance in which IQ and pre-test score were held constant.

TABLE A-7. Pre- and Post-Test Mean Difficulty Indices for Students to Whom Conditional Reasoning Was Deliberately Taught, by Item Groups and Grades.

Item Group	Test	Grade N=	5 27	7 24	9 17	11 26
1	Pre		68	76	74	88
	Post		77	83	78	97
	Change		9	7	4	9
2	Pre		30	21	15	34
	Post		25	28	24	84
	Change		- 5	7	9	50
3	Pre		24	24	29	41
	Post		35	35	31	94
	Change		11	11	2	53
4	Pre		51	68	59	71
	Post		49	74	63	82
	Change		- 2	6	4	11
5	Pre		56	73	68	87
	Post		67	80	65	91
	Change		11	7	- 3	4
6	Pre		44	71	58	67
	Post		65	67	37	86
	Change		21	- 4	-21	19
7	Pre		15	15	28	53
	Post		21	20	33	80
	Change		6	5	5	27
8	Pre		71	83	71	88
	Post		63	77	79	80
	Change		- 8	- 6	8	- 8
9	Pre		70	83	80	93
	Post		72	91	84	80
	Change		2	8	4	-13
10	Pre		50	76	68	88
	Post		48	78	62	91
	Change		- 2	2	- 6	3
11	Pre		47	72	62	72
	Post		48	65	49	71
	Change		1	- 7	-13	- 1
12	Pre		31	16	28	18
	Post		30	13	10	62
	Change		- 1	- 3	-18	44

~~Note: In the necessary condition column a minus sign in front of a change indicates improvement, since a minus sign there shows that fewer on the post-test than on the pre-test failed to meet the necessary condition.~~

TABLE A-8. Pre- and Post-Tests Percentages of LDT's Who Met the Sufficient Condition for Mastery, and Percentages Who Failed to Meet the Necessary Condition for Mastery of Principles of Conditional Reasoning, By Item Group and Grade.

		Meeting the Sufficient Condition				Failing to Meet the Necessary Condition			
Item	Grade	5	7	9	11	5	7	9	11
Group	Test	27	24	17	26	27	24	17	26
1	Pre	48	54	53	81	04	17	24	08
	Post	63	59	53	96	22	13	12	00
	Change	15	5	0	15	18	- 4	-12	- 8
2	Pre	04	00	00	12	85	91	100	73
	Post	00	00	00	77	59	83	94	12
	Change	- 4	0	0	65	-26	- 8	- 6	-61
3	Pre	04	00	06	08	63	96	88	81
	Post	04	17	00	81	96	74	82	12
	Change	0	17	- 6	73	33	-22	- 6	-69
4	Pre	19	59	23	50	74	21	53	23
	Post	26	50	24	77	63	25	47	15
	Change	7	- 9	1	27	-11	4	- 6	- 8
5	Pre	19	61	41	81	41	21	35	12
	Post	44	70	47	83	33	21	41	4
	Change	25	9	6	2	- 8	0	6	- 8
6	Pre	30	57	18	42	67	25	41	31
	Post	41	35	06	81	33	38	76	12
	Change	11	-22	-12	39	-34	13	35	-19
7	Pre	04	00	06	35	96	92	94	58
	Post	04	09	12	77	93	79	82	23
	Change	0	9	6	42	- 3	-13	-12	-35
8	Pre	37	65	53	88	41	21	24	00
	Post	37	48	65	77	37	17	18	15
	Change	0	-17	12	-11	- 4	- 4	- 6	15
9	Pre	52	75	71	96	22	08	12	00
	Post	56	88	76	81	22	04	12	08
	Change	4	13	5	-15	0	- 4	0	8
10	Pre	22	63	47	81	52	17	41	08
	Post	19	59	41	89	52	17	47	12
	Change	- 3	- 4	- 6	8	0	0	6	4
11	Pre	07	50	29	50	56	29	41	23
	Post	26	46	18	50	56	37	59	27
	Change	19	- 4	-11	0	0	8	18	4
12	Pre	07	04	06	00	78	92	88	100
	Post	04	00	-00	50	85	96	100	35
	Change	- 3	- 4	- 6	50	7	4	12	-65

Note: In the necessary conditon columns a minus sign in front of a change indicates improvement, since a minus sign there shows that fewer on the post-test than on the pre-test failed to meet the necessary condition.

TABLE A-9. Pre- and Post-Test Mean Difficult Indices for Students to Whom Class Reasoning Was Deliberately Taught, by Item Groups and Grades.

Item Group	Test	Grade N=	4 25	6 25	8 27	10 22	12 24
1	Pre		76	91	95	96	99
	Post		64	97	93	96	98
	Change		-12	6	- 2	0	- 1
2	Pre		68	74	72	84	89
	Post		70	86	90	90	98
	Change		2	12	18	6	9
3	Pre		34	45	51	71	85
	Post		38	56	72	84	92
	Change		4	11	21	13	7
4	Pre		49	41	49	61	77
	Post		37	55	51	80	78
	Change		-12	14	2	19	1
5	Pre		65	69	78	80	88
	Post		69	80	87	84	95
	Change		4	11	9	4	7
6	Pre		54	82	92	81	91
	Post		61	84	88	85	93
	Change		7	2	- 4	4	2
7	Pre		36	41	40	56	70
	Post		34	47	59	75	77
	Change		- 2	6	19	19	7
8	Pre		39	42	54	64	83
	Post		34	50	67	84	90
	Change		- 5	8	13	20	7
9	Pre		49	54	70	68	78
	Post		53	69	65	73	86
	Change		4	15	- 5	5	8
10	Pre		47	56	61	59	63
	Post		39	53	55	56	73
	Change		- 8	- 3	- 6	- 3	10
11	Pre		46	59	81	74	88
	Post		43	65	83	78	85
	Change		- 3	6	2	4	- 3
12	Pre		46	43	77	72	81
	Post		58	70	79	83	94
	Change		12	27	2	11	13

~~Note:~~ ~~In the necessary condition column a minus sign in front of a change indicates improvement, since a minus sign there shows that fewer on the post-test than on the pre-test failed to meet the necessary condition~~.

TABLE A-10. Pre- and Post-Test Percentages of Students Who Were Deliberately Taught Class Reasoning Who Met the Sufficient Condition For Mastery and Percentages Who Failed to Meet the Necessary Condition, by Item Group and Grade

			Meeting The Sufficient Condition					Failing to Meet The Necessary Condition				
Item		Grade	4	6	8	10	12	4	6	8	10	12
Group	Test	N=	25	25	27	22	24	25	25	27	22	24
1	Pre		62	84	100	96	100	23	00	00	00	00
	Post		50	100	89	96	96	39	00	00	00	00
	Change		-12	16	-11	0	- 4	16	0	0	0	0
2	Pre		42	44	70	82	84	31	20	04	05	04
	Post		58	76	89	86	100	35	04	04	09	00
	Change		16	32	19	4	16	4	-16	0	4	- 4
3	Pre		04	16	30	41	79	81	68	59	32	08
	Post		04	44	56	77	84	89	40	22	14	04
	Change		0	28	26	36	5	8	-28	-37	-18	- 4
4	Pre		12	16	15	32	59	62	80	63	46	21
	Post		08	28	26	59	84	65	44	56	18	08
	Change		- 4	12	11	27	25	3	-36	- 7	-28	-13
5	Pre		46	36	63	73	88	31	20	04	09	04
	Post		42	64	81	82	96	19	12	07	09	00
	Change		- 4	28	18	9	8	-12	- 8	3	0	- 4
6	Pre		27	76	93	77	88	54	08	04	09	04
	Post		39	72	93	86	88	46	08	04	05	04
	Change		12	- 4	0	9	0	- 8	0	0	- 4	0
7	Pre		00	16	22	23	50	89	80	63	55	36
	Post		08	32	37	46	63	81	64	52	27	12
	Change		8	16	15	23	13	- 8	-16	-11	-28	-25
8	Pre		04	08	26	36	71	77	60	56	41	16
	Post		00	20	44	68	79	89	60	30	14	04
	Change		- 4	12	18	32	8	12	0	-26	-27	-13
9	Pre		27	16	41	41	50	59	56	19	32	21
	Post		27	48	30	55	79	48	36	30	23	13
	Change		0	32	-11	14	29	-11	-20	11	- 9	- 8
10	Pre		12	24	37	32	33	59	52	37	55	46
	Post		12	20	19	36	59	74	64	56	55	25
	Change		0	- 4	-18	4	26	15	12	19	0	-21
11	Pre		19	36	67	46	79	55	52	15	18	08
	Post		15	36	74	64	75	67	36	15	14	17
	Change		- 4	0	7	18	- 4	12	-26	0	- 4	9
12	Pre		35	20	67	59	67	52	64	11	14	08
	Post		42	64	78	73	88	48	28	15	00	00
	Change		7	44	11	14	21	- 4	-36	4	-14	- 8

Note: In the necessary condition columns a minus sign in front of a change indicates improvement, since a minus sign there shows that fewer on the post-test than on the pre-test failed to meet the necessary condition.

TABLE A-11. Conditional Reasoning Weights and Constants for Multiple Regression Equations Making Use of Pre-Test.

	Grade	CA	IQ	SES	Sex	Total Pre-Test Score	Corresponding Pre-Test Score	Constant
Total Score	7.826	-.4261	.1179	-1.531	.495	.7480		18.62
Component								
CF	2.760	-.1424	.0385	-.460	-.652	-.0584	.970	6.86
SY	.845	-.0421	.0189	-.174	.419	.0803	-.127	2.37
SU	.899	-.0372	.0084	-.294	.503	.0614	-.120	2.77
Item Group								
1	.254	-.0166	.0155	-.056	.173	.0290	.011	1.94
2	1.307	-.0647	.0035	-.0914	-.018	-.0113	.614	1.68
3	1.379	-.0809	.0062	-.057	-.472	.0003	.133	5.15
4	- .001	.0074	.0064	-.127	.423	.0384	.314	- 1.26
5	.008	-.0039	.0190	-.082	-.275	.0367	.324	.39
6	.123	-.0193	.0163	-.215	.747	.0761	.005	3.81
7	.648	-.0215	-.0142	.018	.280	.0137	.406	.51
8	.088	.0093	.0202	-.097	.082	.0282	.129	- .540
9	- .271	.0173	.0224	-.097	-.453	.0192	.016	.82
10	.177	-.0018	.0384	-.001	-.249	.0239	.305	- 3.49
11	- .322	.0176	.0123	.006	.638	.0757	.140	- 3.61
12	1.489	-.0954	-.0245	-.254	-.090	.0037	.029	9.05

TABLE A-12. Conditional Reasoning Weights and Constants for Multiple Regression Equations Not Using Pre-Test Scores.

	Grade	CA	IQ	SES	Sex	Constant
Total Score	.116	-.5330	.4074	-1.332	.592	10.33
Component						
CF	5.327	-.2557	.1770	- .457	-.495	11.21
SY	1.216	-.0531	.0443	- .169	.462	1.55
SU	1.211	-.0501	.0280	- .280	.483	2.58
Item Group						
1	.402	-.0192	.0305	-.046	.211	1.38
2	1.640	-.0905	.0016	-.080	-.489	5.20
3	1.432	-.0841	.0060	-.059	-.558	5.66
4	.152	.0100	.0362	-.108	.464	-3.30
5	.263	-.0074	.0147	-.055	-.083	- .76
6	.512	-.0302	.0134	-.196	.757	2.96
7	1.110	-.0430	-.0132	.069	.126	1.70
8	.113	.0033	.0360	-.085	.085	- .96
9	-.113	.0123	.0347	-.092	-.465	.75
10	.361	.0020	.0601	.010	-.279	-4.88
11	.056	.0098	.0469	.028	.728	-5.07
12	1.536	-.1003	-.0308	-.261	-.017	10.72

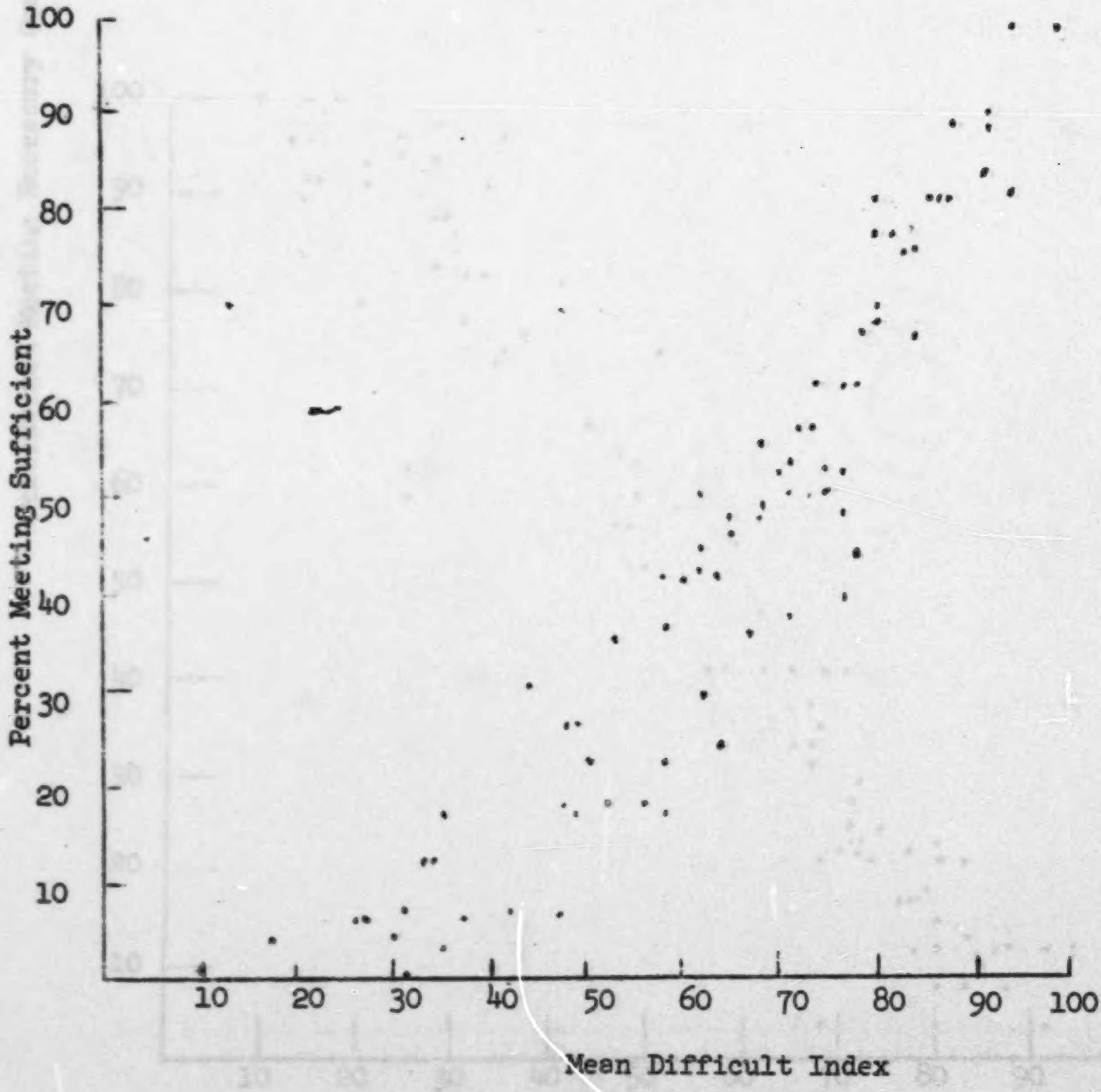
TABLE A-13. Class Reasoning Weights and Constants for Multiple Regression Equations Making Use of Pre-Test.

	Grade	CA	IQ	SES	Sex	Total Pre-Test Score	Corresponding Pre-Test Score	Constant
Total Score	-.778	.2014	.3467	.767	-1.250	.6209		-39.46
Component								
CF	-.049	.0450	.1555	.353	- .221	.0885	.415	-10.06
SY	-.678	.0931	.0230	-.079	.237	.0289	.154	- 7.09
SU	.160	.0003	.0501	.197	- .154	.0860	.104	- 6.06
Item Group								
1	.040	-.0009	.0212	.121	- .007	.0114	.668	- 2.21
2	.337	-.0257	.0254	.035	.025	.0242	.089	1.54
3	-.112	.0229	-.0011	-.052	- .220	.0173	.407	- .57
4	-.394	.0540	.0053	-.078	- .036	-.0000	.396	- 3.58
5	-.086	.0124	.0248	.002	.270	.0095	.170	- 1.08
6	-.246	.0251	.0286	.086	.275	.0144	.350	- 3.53
7	-.045	.0077	.0236	-.006	- .181	.0304	.373	- 3.01
8	.057	.0135	.0104	.016	- .084	-.0241	.430	- .47
9	-.055	.0089	.0321	.092	- .098	.0286	-.007	- 2.73
10	-.277	.0329	.0353	.107	- .226	.0019	.297	- 5.31
11	-.092	.0124	.0302	.079	- .123	.0359	.149	- 3.71
12	.098	-.0051	.0128	.095	- .175	.0294	.144	.77

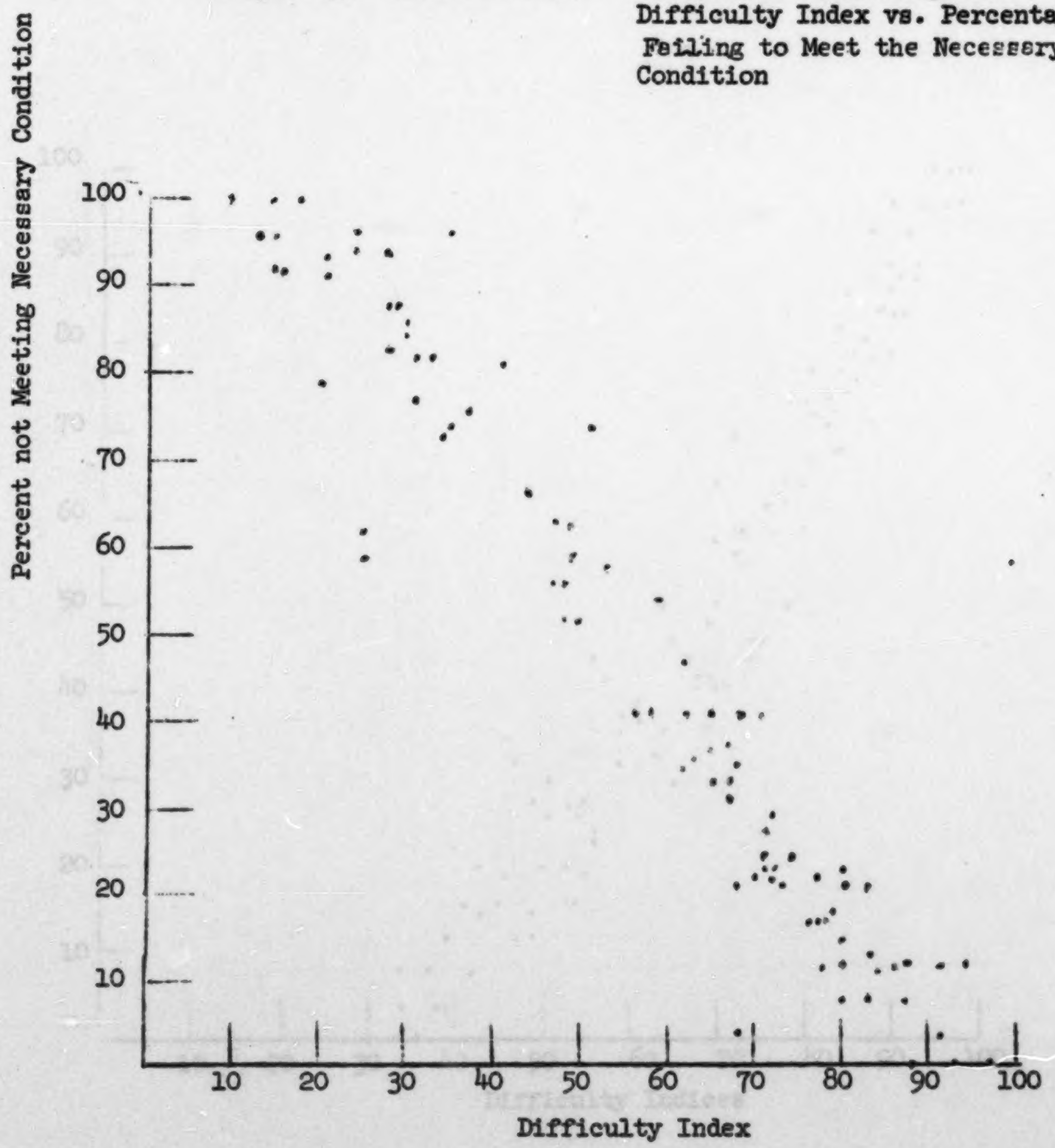
TABLE A-14. Class Reasoning Weights and Constants for Multiple Regression Equations Not Using Pre-Test Scores.

	Grade	CA	IQ	SES	Sex	Constant
Total Score	.741	.2405	.6369	.593	-2.221	-50.54
Component						
CF	.636	.6191	.2835	.310	-.841	-12.43
SY	-.527	.0914	.0423	-.094	-.274	- 7.57
SU	.363	.0095	.0970	.172	-.299	- 8.17
Item Group						
1	.246	-.0107	.0450	.103	-.003	- 5.78
2	.384	-.0224	.0396	.037	-.006	.95
3	-.057	.0335	.0202	-.062	-.312	- 2.60
4	-.215	.0448	.0074	-.134	-.191	- 1.97
5	-.057	.0145	.0316	.000	.236	- 1.04
6	-.072	.0162	.0483	.83	.023	- 3.53
7	.169	.0049	.0441	-.016	-.221	- 3.56
8	.228	.0147	.0127	.011	-.278	- 1.47
9	.015	.0106	.0453	.084	-.143	- 3.24
10	-.355	.0427	.0436	.125	-.341	- 6.06
11	-.013	.0181	.0547	.074	-.134	- 5.21
12	.200	-.0041	.0321	.082	-.253	.15

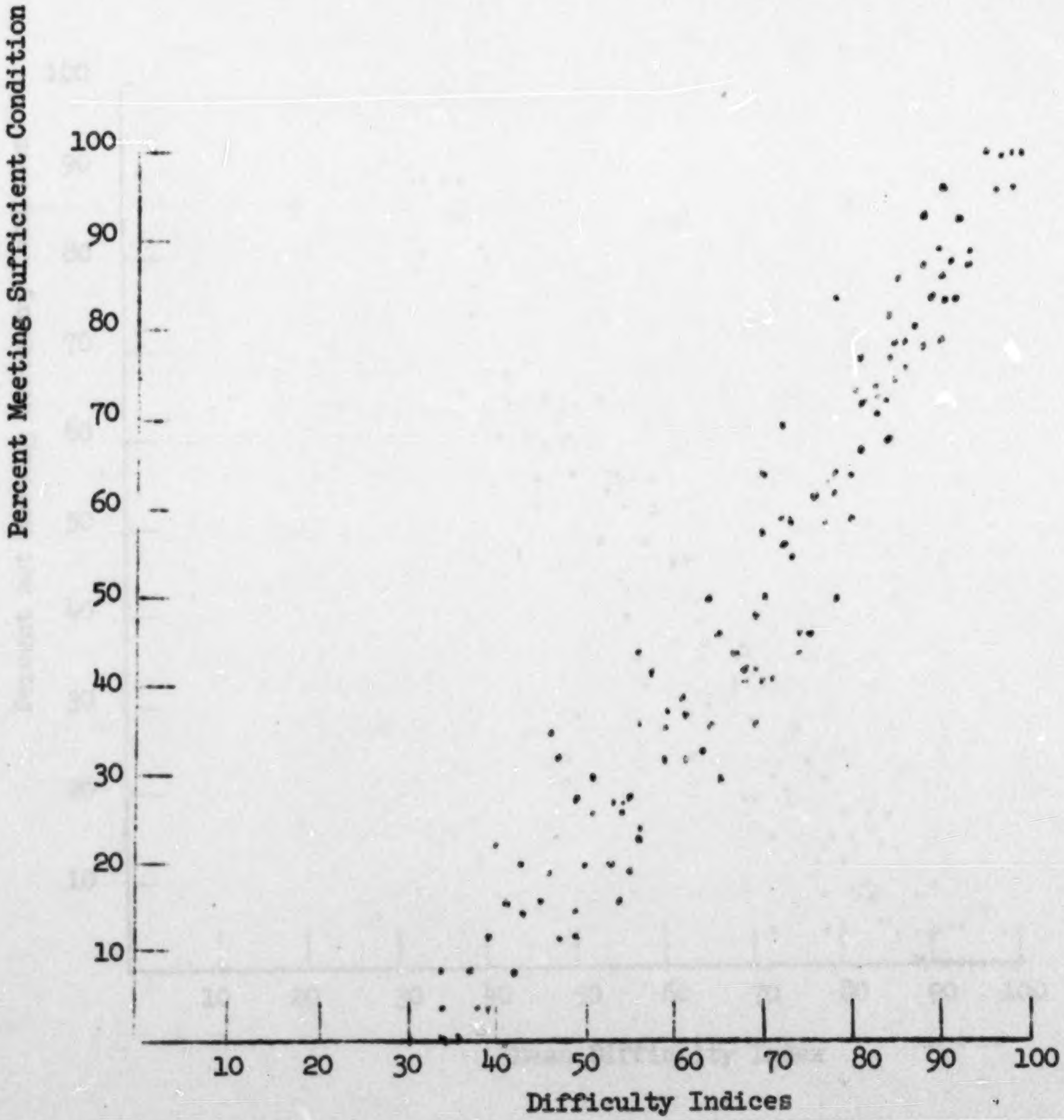
Graph A-1 Plotted Points: Conditional Reasoning Mean Difficulty Index vs. Percentage Meeting the Sufficient Condition



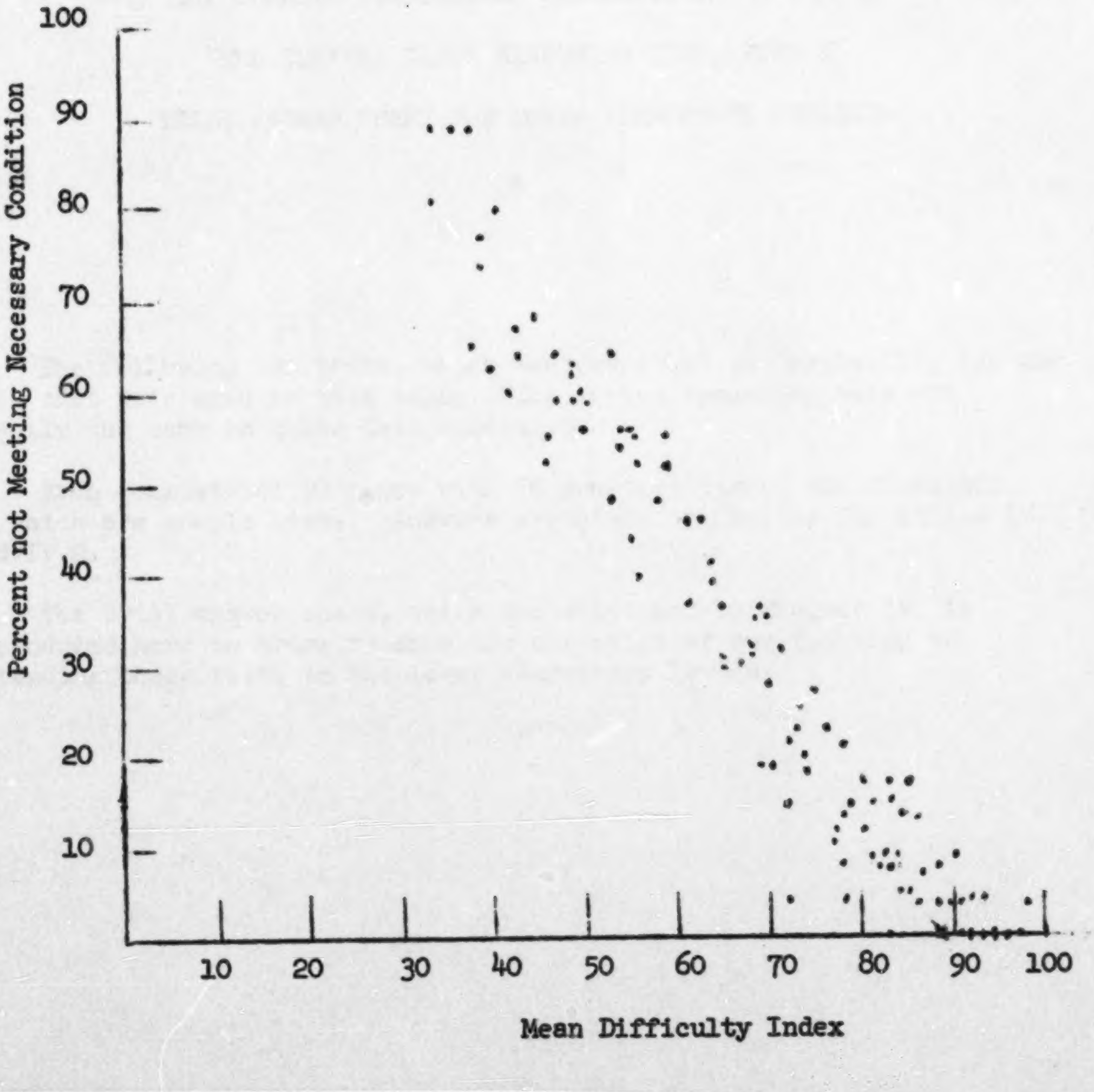
Graph A-2. Plotted Points: Conditional Reasoning Mean Difficulty Index vs. Percentage Failing to Meet the Necessary Condition



Graph A-3. Plotted Points: Class Reasoning Mean Difficulty Index vs. Percentage Meeting the Sufficient Condition.



Graph A-4. Plotted Points: Class Reasoning Mean Difficulty Index vs. Percentage Failing to Meet the Necessary Condition.



APPENDIX B. THE CORNELL DEDUCTION TESTS:

"THE CORNELL CONDITIONAL REASONING TEST, FORM X"

"THE CORNELL CLASS REASONING TEST, FORM X"

TRIAL ANSWER SHEET FOR LOWER ELEMENTARY STUDENTS

The following two tests, which are described in Chapter IV, are the ones that were used in this study. The copies appearing here are exactly the same as those that were used.

Each consists of 22 pages with 78 numbered items, the first six of which are sample items. Answers are given in Chapter IV, Tables IV-1 and IV-2.

The trial answer sheet, which was mentioned in Chapter IV, is reproduced here in order to show the direction of our thinking in extending these tests to the lower elementary levels.

Cornell Critical Thinking Test Series

THE CORNELL CONDITIONAL-REASONING TEST, FORM X

by

Robert H. Ennis
William L. Gordiner
John Guzzetta
Richard Morrow
Dieter Paulus
Lucille Ringel

Fill in the blanks when you are asked to do so:

Print your last name only ______________________

Print your first and middle names ______________________

Your age on your last birthday ______________ years

Your date of birth: month____ day____ year ____

Your grade ______________________

Your school ______________________

Your regular teacher at this time ______________________

Today's date: month____ day____ year ____

Do not write in this space:

General directions:

This is a test to see how well you do a particular kind of thinking. We call it "conditional reasoning". You will see that you already do some of this kind of thinking. The sample questions make clear what is expected.

DO NOT GUESS WILDLY. There is a scoring penalty for guessing wrong. If you think you have the answer, but are not sure, mark that answer. But if you have no idea, then skip the question.

There are 6 sample questions, then 72 others. You should work as quickly as you can, but do not rush. This is not a speed test. Once you do the samples, you will be able to move right along.

DO NOT TURN THE PAGE UNTIL YOUR EXAMINER TELLS YOU TO DO SO.

© 1964 by R. H. Ennis

Published by Cornell Critical Thinking Project, Stone Hall, Ithaca, N.Y.

Answering the questions:

In answering each question, use only what you are told in that question. In order to do this, you should imagine that your mind is blank, because some of the things you are told are obviously false. Even so, you should suppose that they are true--for that question only.

You will be given one or more sentences with which to think. You will then be given another sentence, about which you must decide, using only what you were told.

There are three possible answers. This is what they mean:

A. YES It must be true.
B. NO It can't be true.
C. MAYBE It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

The meaning of the possible answers is given at the top of each page to help you remember. Each question has only one correct answer.

Mark your answers on this booklet by drawing a circle around the right answer. Remember: If you have no idea what the answer is, skip the question and go on to the next. Do not guess wildly, but if you think you know, then answer the question.

Sample questions:

Read the first question and see how it is marked.

1. Suppose you know that

Bill is next to Sam.

Then would this be true?

Sam is next to Bill.

1. A. [✓ YES]
B. NO
C. MAYBE

The correct answer is A, "YES". If Bill is next to Sam, then Sam must be next to Bill. It must be true, so a circle is drawn around "YES".

Here is another sample. This time you circle the answer.

2. Suppose you know that

The sparrow is over the hawk.

Then would this be true?

The hawk is over the sparrow.

2. A. YES
B. NO
C. MAYBE

You should have circled B, "NO". If the sparrow is over the hawk, then the hawk can't be over the sparrow. It can't be true.

Here is a reminder of the meaning of the possible answers:

A. YES It must be true.
B. NO It can't be true.
C. MAYBE It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

Circle the answer to this next sample. Be careful:

3. Suppose you know that

Jane is standing near Betsy.

Then would this be true?

Betsy is standing near Jane.

3. A. YES
B. NO
C. MAYBE

The correct answer is C, "MAYBE". Even if Jane is standing near Betsy, Betsy might be sitting. Betsy might be standing near Jane, but she might be sitting near Jane, or something else. You were not told enough to be certain about it, so "MAYBE" is the answer.

Circle the answer to this next sample question. Remember that your mind is supposed to be blank at the beginning of each question.

4. Suppose you know that

California is near New York.

Then would this be true?

New York is near California.

4. A. YES
B. NO
C. MAYBE

The correct answer is A, "YES", even though New York and California are not really near to each other. If California were near to New York, then New York would be near to California. It would have to be true.

Remember: You should suppose that what you are told is true -- for the question you are answering.

Here is a reminder of the meaning of the possible answers:

A. YES It must be true.
B. NO It can't be true.
C. MAYBE It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

So far in the sample questions you were told only one thing. In this one you are told two things. Circle your answer.

5. Suppose you know that

The pit is inside the mouth of the fox.
The cherry is inside the mouth of the fox.

Then would this be true?

The pit is inside the cherry.

5. A. YES
B. NO
C. MAYBE

The correct answer is C, "MAYBE". All you are told is that the pit and the cherry are both in the mouth of the fox. There is no way to be certain whether the pit is in the cherry or not.

Here is the last sample question. This time the letters "X" and "Y" are used. They can stand for anything you like. Circle your answer:

6. Suppose you know that

X is next to Y.

Then would this be true?

Y is next to X.

6. A. YES
B. NO
C. MAYBE

The correct answer is A, "YES", no matter what X and Y stand for. If X is next to Y, then Y must be next to X.

Now that you have done the practice questions you probably understand what is expected. If you have any questions, ask them now.

DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO.

Here is a reminder of the meaning of the possible answers:

A. YES It must be true.
B. NO It can't be true.
C. MAYBE It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

---

7. Suppose you know that

If the hat on the table is blue, then it belongs to Joan.
The hat on the table is blue.

Then would this be true?

The hat on the table belongs to Joan.

7. A. YES
B. NO
C. MAYBE

---

8. Suppose you know that

If the car in the parking lot is Mr. Smith's, then it is blue.
The car in the parking lot is not blue.

Then would this be true?

The car in the parking lot is Mr. Smith's.

8. A. YES
B. NO
C. MAYBE

---

9. Suppose you know that

If Tom lives in the white house, then his last name is Smith.
Tom does not live in the white house.

Then would this be true?

Tom's last name is not Smith.

9. A. YES
B. NO
C. MAYBE

---

10. Suppose you know that

Harry is on the football team only if he has his mother's permission.
Harry is on the football team.

Then would this be true?

Harry has his mother's permission.

10. A. YES
B. NO
C. MAYBE

Here is a reminder of the meaning of the possible answers:

A. YES It must be true.
B. NO It can't be true.
C. MAYBE It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

---

11. Suppose you know that

If Mary lives in the white house, then her last name is Brown.
Mary's last name is Brown.

Then would this be true?

Mary lives in the white house.

11. A. YES
B. NO
C. MAYBE

12. Suppose you know that

John is in the kitchen only if there is food in the kitchen.
There is no food in the kitchen.

Then would this be true?

John is in the kitchen.

12. A. YES
B. NO
C. MAYBE

13. Suppose you know that

If the automobile in the parking lot belongs to Mr. Brown, then it is black.
The automobile in the parking lot doesn't belong to Mr. Brown.

Then would this be true?

The automobile isn't black.

13. A. YES
B. NO
C. MAYBE

14. Suppose you know that

Joe's bicycle is not working today.
If Joe's bicycle is not working, then he has to walk to school.

Then would this be true?

Joe has to walk to school today.

14. A. YES
B. NO
C. MAYBE

Here is a reminder of the meaning of the possible answers:

A. YES It must be true.
B. NO It can't be true.
C. MAYBE It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

---

15. Suppose you know that

There is an X only if there is a Y.
There is not a Y.

Then would this be true?

There is an X.

15. A. YES
B. NO
C. MAYBE

---

16. Suppose you know that

Dick was not at home yesterday afternoon.
If Dick was not at the football game yesterday afternoon he was at home.

Then would this be true?

Dick was not at the football game yesterday afternoon.

16. A. YES
B. NO
C. MAYBE

---

17. Suppose you know that

Tom may use paints only if he has cleaned up his clay work.
Tom may use paints.

Then would this be true?

Tom has cleaned up his clay work.

17. A. YES
B. NO
C. MAYBE

---

18. Suppose you know that

Fred went to a movie last night.
If Fred does not go to a movie, he feels bad the next day.

Then would this be true?

Fred does not feel bad today.

18. A. YES
B. NO
C. MAYBE

Here is a reminder of the meaning of the possible answers:

A. YES It must be true.
B. NO It can't be true.
C. MAYBE It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

---

19. Suppose you know that

If there is an X, then there is a Y.
There is an X.

Then would this be true?

There is a Y.

19. A. YES
B. NO
C. MAYBE

---

20. Suppose you know that

Mary will be in the school play only if she likes plays.
Mary will be in the school play.

Then would this be true?

Mary does not like plays.

20. A. YES
B. NO
C. MAYBE

---

21. Suppose you know that

Tom is playing ball only if he has a ball glove.
Tom does not have a ball glove.

Then would this be true?

Tom is playing ball.

21. A. YES
B. NO
C. MAYBE

---

22. Suppose you know that

If there is an X, then there is a Y.
There is not a Y.

Then would this be true?

There is an X.

22. A. YES
B. NO
C. MAYBE

Here is a reminder of the meaning of the possible answers:

A. YES It must be true.
B. NO It can't be true.
C. MAYBE It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

23. Suppose you know that

If whales are birds, then they can fly.
Whales aren't birds.

Then would this be true?

Whales can't fly.

23. A. YES
B. NO
C. MAYBE

24. Suppose you know that

If Bill lives on a farm, then he has a pet dog.
Bill has a pet dog.

Then would this be true?

Bill lives on a farm.

24. A. YES
B. NO
C. MAYBE

25. Suppose you know that

Jerry was not asked to play ball.
Jerry is not home only if he was asked to play ball.

Then would this be true?

Jerry is not home.

25. A. YES
B. NO
C. MAYBE

26. Suppose you know that

If Mary lives in the green house, then her last name is Jones.
Mary doesn't live in the green house.

Then would this be true?

Mary's last name is not Jones.

26. A. YES
B. NO
C. MAYBE

Here is a reminder of the meaning of the possible answers:

A. YES It must be true.
B. NO It can't be true.
C. MAYBE It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

---

27. Suppose you know that

If the coat in the closet is brown, then it belongs to Sue.
The coat in the closet is brown.

Then would this be true?

The coat in the closet does not belong to Sue.

27. A. YES
B. NO
C. MAYBE

28. Suppose you know that

There are black cats only if there are pink cats.
There are black cats.

Then would this be true?

There are pink cats.

28. A. YES
B. NO
C. MAYBE

29. Suppose you know that

If the bicycle in the garage is Bob's, then it is red.
The bicycle in the garage is not red.

Then would this be true?

The bicycle in the garage is not Bob's.

29. A. YES
B. NO
C. MAYBE

30. Suppose you know that

If there is an X, then there is a Y.
There is a Y.

Then would this be true?

There is an X.

30. A. YES
B. NO
C. MAYBE

Here is a reminder of the meaning of the possible answers:

A. YES It must be true.
B. NO It can't be true.
C. MAYBE It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

---

31. Suppose you know that

If mice have five legs, then they run faster than horses.
Mice do have five legs.

Then would this be true?

Mice run faster than horses.

31. A. YES
B. NO
C. MAYBE

---

32. Suppose you know that

If Jane fell off her horse, then she hurt herself badly.
Jane hurt herself badly.

Then would this be true?

Jane fell off her horse.

32. A. YES
B. NO
C. MAYBE

---

33. Suppose you know that

The short pencil is not Bill's favorite pencil.
The short pencil is not Bill's favorite, only if it is dull.

Then would this be true?

The short pencil is dull.

33. A. YES
B. NO
C. MAYBE

---

34. Suppose you know that

If there is an X, then there is a Y.
There is not an X.

Then would this be true?

There is not a Y.

34. A. YES
B. NO
C. MAYBE

Here is a reminder of the meaning of the possible answers:

A.	YES	It must be true.
B.	NO	It can't be true.
C.	MAYBE	It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

---

35. Suppose you know that

If John lives in the white house, then his last name is Smith.
John's last name is not Smith.

Then would this be true?

John does live in the white house.

35. A. YES
B. NO
C. MAYBE

---

36. Suppose you know that

Birds can fly only if they can play the piano.
Birds cannot play the piano.

Then would this be true?

Birds can fly.

36. A. YES
B. NO
C. MAYBE

---

37. Suppose you know that

The car will start.
If the temperature is not below freezing, the car will start.

Then would this be true?

The temperature is not below freezing.

37. A. YES
B. NO
C. MAYBE

---

38. Suppose you know that

There is an X only if there is a Y.
There is an X.

Then would this be true?

There is a Y.

38. A. YES
B. NO
C. MAYBE

Here is a reminder of the meaning of the possible answers:

A. YES It must be true.
B. NO It can't be true.
C. MAYBE It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

---

39. Suppose you know that

If dogs have four legs, then they have three eyes.
Dogs don't have three eyes.

Then would this be true?

Dogs do have four legs.

39. A. YES
B. NO
C. MAYBE

---

40. Suppose you know that

If Jean goes to the park, she will see her friend Pat.
Today, Jean is going to the park.

Then would this be true?

Today, Jean will see her friend Pat.

40. A. YES
B. NO
C. MAYBE

---

41. Suppose you know that

If horses are green, then they have two tails.
Horses have two tails.

Then would this be true?

Horses are green.

41. A. YES
B. NO
C. MAYBE

---

42. Suppose you know that

The red pencils belong to Sally only if they are on the table.
The red pencils are not on the table.

Then would this be true?

The red pencils do not belong to Sally.

42. A. YES
B. NO
C. MAYBE

Here is a reminder of the meaning of the possible answers:

A. YES It must be true.
B. NO It can't be true.
C. MAYBE It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

---

43. Suppose you know that

If Paul rides his bike to school, he goes the long way.
Paul rode his bike to school today.
If Paul goes the long way, he gets to school late.

Then would this be true?

Paul was not late for school today.

43. A. YES
B. NO
C. MAYBE

44. Suppose you know that

If the chair is green, then the table is black.

Then would this be true?

If the table is black, then the chair is green.

44. A. YES
B. NO
C. MAYBE

45. Suppose you know that

If there is a blue pencil in the second box, then there is a green pencil in the first box.
If there is a green pencil in the first box, then there is a red pencil in the third box.

Then would this be true?

If there is a blue pencil in the second box, then there is a red pencil in the third box.

45. A. YES
B. NO
C. MAYBE

46. Suppose you know that

If Mrs. Smith entered the flower show, then she entered her roses.

Then would this be true?

If Mrs. Smith didn't enter her roses, then she didn't enter the flower show.

46. A. YES
B. NO
C. MAYBE

Here is a reminder of the meaning of the possible answers:

A. YES It must be true.
B. NO It can't be true.
C. MAYBE It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

---

47. Suppose you know that

Bill will see Audrey, if and only if he goes to Montreal.
Bill will not see Audrey this year.

Then would this be true?

Bill is going to Montreal this year.

47. A. YES
B. NO
C. MAYBE

48. Suppose you know that

If Gary sees Sharon, he goes to Canada.
This winter Gary saw Sharon.
Gary goes skating only if he goes to Canada.

Then would this be true?

This winter Gary went skating.

48. A. YES
B. NO
C. MAYBE

49. Suppose you know that

If there is an A, then there is a B.
If there is a B, then there is a C.

Then would this be true?

If there is an A, then there is a C.

49. A. YES
B. NO
C. MAYBE

50. Suppose you know that

If birds can fly, then they have six legs.

Then would this be true?

If birds don't have six legs, then they can't fly.

50. A. YES
B. NO
C. MAYBE

Here is a reminder of the meaning of the possible answers:

A. YES It must be true.
B. NO It can't be true.
C. MAYBE It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

---

51. Suppose you know that

If the bus goes to town, then it passes the old stone church.
The bus goes to town.
If it passes the old stone church, then it goes over the new bridge.

Then would this be true?

The bus doesn't go over the new bridge.

51. A. YES
B. NO
C. MAYBE

52. Suppose you know that

If the school team loses this game, Brighton High will win the league pennant.
If Joe does not hit a homer on this pitch, the school team will lose this game.

Then would this be true?

If Joe does not hit a homer on this pitch, Brighton High will win the league pennant.

52. A. YES
B. NO
C. MAYBE

53. Suppose you know that

If Jean goes shopping, she goes to Chicago.
Last Saturday Jean went shopping.
Jean visits her aunt only if she goes to Chicago.

Then would this be true?

Last Saturday Jean visited her aunt.

53. A. YES
B. NO
C. MAYBE

54. Suppose you know that

Tom will go skating, if and only if he can borrow Frank's jacket.
Tom is not going skating.

Then would this be true?

Tom can borrow Frank's jacket.

54. A. YES
B. NO
C. MAYBE

Here is a reminder of the meaning of the possible answers:

A. YES It must be true.
B. NO It can't be true.
C. MAYBE It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

---

55. Suppose you know that

If Sam misses the bus, he will walk to school.
If Sam walks to school, he will cross the bridge.

Then would this be true?

If Sam misses the bus, he will cross the bridge.

55. A. YES
B. NO
C. MAYBE

56. Suppose you know that

If Bob did not buy a new baseball glove, then he played basketball today.

Then would this be true?

If Bob did not play basketball today, then he did buy a new baseball glove.

56. A. YES
B. NO
C. MAYBE

57. Suppose you know that

If Bill has an apple in his lunchbox, then Sally has a cracker in her lunchbox.

Then would this be true?

If Sally has a cracker in her lunchbox, then Bill has an apple in his lunchbox.

57. A. YES
B. NO
C. MAYBE

58. Suppose you know that

Betty is going to the movies.
Betty is not going to the movies, if and only if Ann is going to the movies.

Then would this be true?

Ann is going to the movies.

58. A. YES
B. NO
C. MAYBE

Here is a reminder of the meaning of the possible answers:

A. YES It must be true.
B. NO It can't be true.
C. MAYBE It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

---

59. Suppose you know that

If there is an X, then there is a Y.

Then would this be true?

If there is a Y, then there is an X.

59. A. YES
B. NO
C. MAYBE

---

60. Suppose you know that

Elephants are pink, if and only if they are large.
Elephants are not pink.

Then would this be true?

Elephants are large.

60. A. YES
B. NO
C. MAYBE

---

61. Suppose you know that

If there is an X, then there is a Y.

Then would this be true?

If there is not a Y, then there is not an X.

61. A. YES
B. NO
C. MAYBE

---

62. Suppose you know that

If John has the red chalk, then he is making a poster for the play.
John has the red chalk.
If John is making a poster for the play, then he is in the library.

Then would this be true?

John is in the library.

62. A. YES
B. NO
C. MAYBE

Here is a reminder of the meaning of the possible answers:

A. YES It must be true.
B. NO It can't be true.
C. MAYBE It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

---

63. Suppose you know that

That bicycle belongs to John, if and only if it is red.
That bicycle does not belong to John.

Then would this be true?

That bicycle is not red.

63. A. YES
B. NO
C. MAYBE

---

64. Suppose you know that

If a dog can stand on its front legs, then it is a puppy.

Then would this be true?

If a dog is a puppy, then it can stand on its front legs.

64. A. YES
B. NO
C. MAYBE

---

65. Suppose you know that

If there is an X, then there is a Y.
There is an X.
There is a Z only if there is a Y.

Then would this be true?

There is a Z.

65. A. YES
B. NO
C. MAYBE

---

66. Suppose you know that

If Kate is in Mrs. Jones' class, then she is out on the playground.
If Kate is out on the playground, then she is jumping rope.

Then would this be true?

If Kate is in Mrs. Jones' class, then she is jumping rope.

66. A. YES
B. NO
C. MAYBE

Here is a reminder of the possible answers:

A. YES It must be true.
B. NO It can't be true.
C. MAYBE It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

---

67. Suppose you know that

If there is an X, then there is a Y.
There is an X.
If there is a Y, then there is a Z.

Then would this be true?

There is not a Z.

67. A. YES
B. NO
C. MAYBE

68. Suppose you know that

If Jane did not go to the movies yesterday, then she saw her friend Pat.
Jane went to the park yesterday only if she saw her friend Pat.
Jane did not go to the movies yesterday.

Then would this be true?

Jane went to the park yesterday.

68. A. YES
B. NO
C. MAYBE

69. Suppose you know that

If Nancy bought a new dress, then she went to the shop on Main Street.

Then would this be true?

If Nancy didn't go to the shop on Main Street, then she didn't buy a new dress.

69. A. YES
B. NO
C. MAYBE

70. Suppose you know that

If John is not in school, then he has a cold.

Then would this be true?

If John has a cold, then he is not in school.

70. A. YES
B. NO
C. MAYBE

Here is a reminder of the meaning of the possible answers:

A. YES It must be true.
B. NO It can't be true.
C. MAYBE It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

---

71. Suppose you know that

If Sally is writing a report at home, then the library is closed.
Sally is writing a report at home.
Dick is using the classroom dictionary only if the library is closed.

Then would this be true?

Dick is using the classroom dictionary.

71. A. YES
B. NO
C. MAYBE

72. Suppose you know that

If there are no blue pencils in the first box, then there is a green pencil in the second box.
If there is a green pencil in the second box, then there is a red pencil in the third box.
There are no blue pencils in the first box.

Then would this be true?

There are no red pencils in the third box.

72. A. YES
B. NO
C. MAYBE

73. Suppose you know that

If an animal is a turtle, then it can fly.
If an animal can fly, then it has feathers.

Then would this be true?

If an animal is a turtle, then it has feathers.

73. A. YES
B. NO
C. MAYBE

74. Suppose you know that

If there is a yellow marble in the first box, then there is a blue marble in the second box.

Then would this be true?

If there is not a blue marble in the second box, then there is not a yellow marble in the first box.

74. A. YES
B. NO
C. MAYBE

Here is a reminder of the meaning of the possible answers:

A. YES It must be true.
B. NO It can't be true.
C. MAYBE It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

---

75. Suppose you know that

If people have fins, then they live in water.
People have fins.
People can swim only if they live in water.

Then would this be true?

People can swim.

75. A. YES
B. NO
C. MAYBE

76. Suppose you know that

If this animal is a dog, then it can fly.
This animal is a dog.
If an animal can fly, then it has feathers.

Then would this be true?

This animal does not have feathers.

76. A. YES
B. NO
C. MAYBE

77. Suppose you know that

If John is on the volleyball team then he is good at volleyball.

Then would this be true?

If John is good at volleyball, then he is on the volleyball team.

77. A. YES
B. NO
C. MAYBE

78. Suppose you know that

There is a Y, if and only if there is an X.
There is not a Y.

Then would this be true?

There is an X.

78. A. YES
B. NO
C. MAYBE

END OF TEST. GO BACK AND CHECK YOUR ANSWERS.

Cornell Critical Thinking Test Series

THE CORNELL CLASS-REASONING TEST, FORM X

by

Robert H. Ennis
William L. Gardiner
Richard Morrow
Dieter Paulus
Lucille Ringel

Fill in the blanks when you are asked to do so:

Print your last name only ____________________

Print your first and middle names ____________________

Your age on your last birthday ____________________ years

Your date of birth: month _____ day _____ year _____

Your grade ____________________

Your school ____________________

Your regular teacher at this time ____________________

Today's date: month _____ day _____ year _____

Do not write in this space:

General directions:

This is a test to see how well you do a particular kind of thinking. We call it "class reasoning". You will see that you already do some of this kind of thinking. The sample questions make clear what is expected.

DO NOT GUESS WILDLY. There is a scoring penalty for guessing wrong. If you think you have the answer, but are not sure, mark that answer. But if you have no idea, then skip the question.

There are 6 sample questions, then 72 others. You should work as quickly as you can, but do not rush. This is not a speed test. Once you do the samples, you will be able to move right along.

DO NOT TURN THE PAGE UNTIL YOUR EXAMINER TELLS YOU
TO DO SO

ⓒ 1964 by R. H. Ennis

Published by Cornell Critical Thinking Project, Stone Hall, Ithaca, N.Y.

Answering the questions:

In answering each question, use only what you are told in that question. In order to do this, you should imagine that your mind is blank, because some of the things you are told are obviously false. Even so, you should suppose that they are true--for that question only.

You will be given one or more sentences with which to think. You will then be given another sentence, about which you must decide, using only what you were told.

There are three possible answers. This is what they mean:

A. YES It must be true.
B. NO It can't be true.
C. MAYBE It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

The meaning of the possible answers is given at the top of each page to help you remember. Each question has only one correct answer.

Mark your answers on this booklet by drawing a circle around the right answer. Remember: If you have no idea what the answer is, skip the question and go on to the next. Do not guess wildly, but if you think you know, then answer the question.

Sample questions:

Read the first question and see how it is marked.

---

1. Suppose you know that

Bill is next to Sam.

Then would this be true?

Sam is next to Bill.

1. A. [✓ YES]
B. NO
C. MAYBE

The correct answer is A, "YES". If Bill is next to Sam, then Sam must be next to Bill. It must be true, so a circle is drawn around "YES".

Here is another sample. This time you circle the answer.

---

2. Suppose you know that

The sparrow is over the hawk.

Then would this be true?

The hawk is over the sparrow.

2. A. YES
B. NO
C. MAYBE

You should have circled B, "NO". If the sparrow is over the hawk, then the hawk can't be over the sparrow. It can't be true.

Here is a reminder of the meaning of the possible answers:

A. YES It must be true.
B. NO It can't be true.
C. MAYBE It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

Circle the answer to this next sample. Be careful:

3. Suppose you know that

Jane is standing near Betsy.

Then would this be true?

Betsy is standing near Jane.

3. A. YES
B. NO
C. MAYBE

The correct answer is C, "MAYBE". Even if Jane is standing near Betsy, Betsy might be sitting. Betsy might be standing near Jane, but she might be sitting near Jane, or something else. You were not told enough to be certain about it, so "MAYBE" is the answer.

Circle the answer to this next sample question. Remember that your mind is supposed to be blank at the beginning of each question.

4. Suppose you know that

California is near New York.

Then would this be true?

New York is near California.

4. A. YES
B. NO
C. MAYBE

The correct answer is A, "YES", even though New York and California are not really near to each other. If California were near to New York, then New York would be near to California. It would have to be true.

Remember: You should suppose that what you are told is true -- for the question you are answering.

Here is a reminder of the meaning of the possible answers:

A. YES It must be true.
B. NO It can't be true.
C. MAYBE It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

So far in the sample questions you were only told one thing. In this one you are told two things. Circle your answer.

---

5. Suppose you know that

The pit is inside the mouth of the fox.
The cherry is inside the mouth of the fox.

Then would this be true?

The pit is inside the cherry.

5. A. YES
B. NO
C. MAYBE

The correct answer is C, "MAYBE". All you are told is that the pit and the cherry are both in the mouth of the fox. There is no way to be certain whether the pit is in the cherry or not.

Here is the last sample question. This time the letters "X" and "Y" are used. They can stand for anything you like. Circle your answer:

---

6. Suppose you know that

X is next to Y.

Then would this be true?

Y is next to X.

6. A. YES
B. NO
C. MAYBE

The correct answer is A, "YES", no matter what X and Y stand for. If X is next to Y, then Y must be next to X.

Now that you have done the practice questions you probably understand what is expected. If you have any questions, ask them now.

DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO.

Here is a reminder of the meaning of the possible answers.

A. YES It must be true.
B. NO It can't be true.
C. MAYBE It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

---

7. Suppose you know that

All the cars in the garage are Mr. Smith's.
All Mr. Smith's cars are Fords.

Then would this be true?

All of the cars in the garage are Fords.

7. A. YES
B. NO
C. MAYBE

8. Suppose you know that

All John's pencils are blue.

Then would this be true?

At least some of John's pencils are not blue.

8. A. YES
B. NO
C. MAYBE

9. Suppose you know that

All the books about sailing are Bill's.
All the green books are Bill's.

Then would this be true?

At least some of the green books are about sailing.

9. A. YES
B. NO
C. MAYBE

10. Suppose you know that

None of Jane's dolls have hats.

Then would this be true?

None of the dolls that have hats are Jane's.

10. A. YES
B. NO
C. MAYBE

Here is a reminder of the meaning of the possible answers.

A. YES It must be true.
B. NO It can't be true.
C. MAYBE It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

---

11. Suppose you know that

All the red books are John's.

Then would this be true?

All John's books are red.

11. A. YES
B. NO
C. MAYBE

---

12. Suppose you know that

All of Mary's books are about horses.
None of the books on the shelf are about horses.

Then would this be true?

At least some of Mary's books are on the shelf.

12. A. YES
B. NO
C. MAYBE

---

13. Suppose you know that

All Jean's pencils are red.
All the pencils on the table are red.

Then would this be true?

At least some of the pencils on the table are Jean's.

13. A. YES
B. NO
C. MAYBE

---

14. Suppose you know that

At least some of the children in the Martin family take out books from the library.
All people who take out books from the library have library cards.

Then would this be true?

At least some of the children in the Martin family have library cards.

14. A. YES
B. NO
C. MAYBE

Here is a reminder of the meaning of the possible answers.

A. YES It must be true.
B. NO It can't be true.
C. MAYBE It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

---

15. Suppose you know that

All X's are Y's.
No Z's are Y's.

Then would this be true?

At least some X's are Z's.

15. A. YES
B. NO
C. MAYBE

16. Suppose you know that

At least some of Fred's pencils are green.

Then would this be true?

None of Fred's pencils are green.

16. A. YES
B. NO
C. MAYBE

17. Suppose you know that

None of Sue's books are about animals.

Then would this be true?

None of the books about animals are Sue's.

17. A. YES
B. NO
C. MAYBE

18. Suppose you know that

At least some of Kate's pencils are blue.
All the pencils in the box are blue.

Then would this be true?

At least some of Kate's pencils are in the box.

18. A. YES
B. NO
C. MAYBE

Here is a reminder of the meaning of the possible answers.

A.	YES	It must be true.
B.	NO	It can't be true.
C.	MAYBE	It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

19. Suppose you know that

All Z's are Y's.
All Y's are X's.

Then would this be true?

All Z's are X's.

19. A. YES
B. NO
C. MAYBE

20. Suppose you know that

None of the fifth grade boys are on the football team.
John is a fifth grade boy.

Then would this be true?

John is not on the football team.

20. A. YES
B. NO
C. MAYBE

21. Suppose you know that

All the members of the school band have been in Boston.
No one in Frank's class has been in Boston.

Then would this be true?

At least some members of the school band are in Frank's class.

21. A. YES
B. NO
C. MAYBE

22. Suppose you know that

All X's are Y's.

Then would this be true?

At least some X's are not Y's.

22. A. YES
B. NO
C. MAYBE

Here is a reminder of the meaning of the possible answers.

A.	YES	It must be true.
B.	NO	It can't be true.
C.	MAYBE	It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

23. Suppose you know that

All boys are painters.
All children are painters.

Then would this be true?

At least some children are boys.

23. A. YES
B. NO
C. MAYBE

24. Suppose you know that

All the second grade children are out on the playground.

Then would this be true?

All the children out on the playground are in the second grade.

24. A. YES
B. NO
C. MAYBE

25. Suppose you know that

At least some of the books on the table are about stars.
None of Bob's books are about stars.

Then would this be true?

All of the books on the table are Bob's.

25. A. YES
B. NO
C. MAYBE

26. Suppose you know that

All the boys in John's class are football players.
Fred is a football player.

Then would this be true?

Fred is not in John's class.

26. A. YES
B. NO
C. MAYBE

Here is a reminder of the meaning of the possible answers.

A. YES It must be true.
B. NO It can't be true.
C. MAYBE It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

---

27. Suppose you know that

All the pets of the Greens' won some prize in the pet show.
Fido is one of the Greens' pets.

Then would this be true?

Fido won a prize in the pet show.

27. A. YES
B. NO
C. MAYBE

---

28. Suppose you know that

No animals are dogs.

Then would this be true?

No dogs are animals.

28. A. YES
B. NO
C. MAYBE

---

29. Suppose you know that

Eileen is one of the children on the playground.

Then would this be true?

Eileen is not one of the children on the playground.

29. A. YES
B. NO
C. MAYBE

---

30. Suppose you know that

All X's are Y's.

Then would this be true?

All Y's are X's.

30. A. YES
B. NO
C. MAYBE

Here is a reminder of the meaning of the possible answers.

A. YES It must be true.
B. NO It can't be true.
C. MAYBE It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

---

31. Suppose you know that

All cats can fly.
All animals that can fly are black.

Then would this be true?

All cats are black.

31. A. YES
B. NO
C. MAYBE

32. Suppose you know that

All the things in the trunk are Bill's.
The brown baseball bat is Bill's.

Then would this be true?

The brown baseball bat is in the trunk.

32. A. YES
B. NO
C. MAYBE

33. Suppose you know that

None of Bob's books are on the table, but there are books on the table.

Then would this be true.

At least some of the books on the table are not Bob's.

33. A. YES
B. NO
C. MAYBE

34. Suppose you know that

All X's are Y's.
All Z's are Y's.

Then would this be true?

At least some Z's are X's.

34. A. YES
B. NO
C. MAYBE

Here is a reminder of the meaning of the possible answers.

A.	YES	It must be true.
B.	NO	It can't be true.
C.	MAYBE	It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

35. Suppose you know that

All Mary's pencils are yellow.

Then would this be true?

At least some of Mary's pencils are not yellow.

35. A. YES
B. NO
C. MAYBE

36. Suppose you know that

All pencils are heavy.
Nothing made of wood is heavy.

Then would this be true?

At least some pencils are made of wood.

36. A. YES
B. NO
C. MAYBE

37. Suppose you know that

At least some of the green pencils are Dick's.

Then would this be true?

All Dick's pencils are green.

37. A. YES
B. NO
C. MAYBE

38. Suppose you know that

No X's are Y's.

Then would this be true?

No Y's are X's.

38. A. YES
B. NO
C. MAYBE

Here is a reminder of the meaning of the possible answers.

A. YES It must be true.
B. NO It can't be true.
C. MAYBE It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

---

39. Suppose you know that

All dogs are brown.

Then would this be true?

At least some dogs are not brown.

39. A. YES
B. NO
C. MAYBE

---

40. Suppose you know that

All the cookies Jane made for the fair had nuts in them.
All the cookies with nuts in them were sold.

Then would this be true.

All the cookies Jane made for the fair were sold.

40. A. YES
B. NO
C. MAYBE

---

41. Suppose you know that

All brown animals have four legs.

Then would this be true?

All animals with four legs are brown.

41. A. YES
B. NO
C. MAYBE

---

42. Suppose you know that

All members of the football team weigh over 150 pounds.
Henry does not weigh over 150 pounds.

Then would this be true?

Henry is on the football team.

42. A. YES
B. NO
C. MAYBE

Here is a reminder of the meaning of the possible answers.

A. YES It must be true.
B. NO It can't be true.
C. MAYBE It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

---

43. Suppose you know that

All of John's candy is in the box.
All of the candy that is not chocolate is also not in the box.

Then would this be true?

At least some of John's candy is not chocolate.

43. A. YES
B. NO
C. MAYBE

---

44. Suppose you know that

All the papers in the box are torn.
None of John's papers are in the box.

Then would this be true?

None of John's papers are torn.

44. A. YES
B. NO
C. MAYBE

---

45. Suppose you know that

All of the boys are singing.

Then would this be true?

All of the people who are not singing are also not boys.

45. A. YES
B. NO
C. MAYBE

---

46. Suppose you know that

All the math homework is due today.
None of Joan's homework is due today.
All the homework for Mr. Miller's class is math homework.

Then would this be true?

None of Joan's homework is for Mr. Miller's class.

46. A. YES
B. NO
C. MAYBE

Here is a reminder of the meaning of the possible answers.

A.	YES	It must be true.
B.	NO	It can't be true.
C.	MAYBE	It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

---

47. Suppose you know that

All the pencils in the box are green.
All Sue's pencils are sharp.
All the green pencils are Sue's.

Then would this be true?

At least some of the pencils in the box are not sharp.

47. A. YES
B. NO
C. MAYBE

---

48. Suppose you know that

None of my shirts are wool.
None of the shirts hanging up in the closet are wool.

Then would this be true?

At least some of my shirts are hanging up in the closet.

48. A. YES
B. NO
C. MAYBE

---

49. Suppose you know that

All X's are Y's.

Then would this be true?

All things that are not Y's are also not X's.

49. A. YES
B. NO
C. MAYBE

---

50. Suppose you know that

All four-legged animals can fly.
No horses can fly.
All fast runners are four-legged animals.

Then would this be true?

No horses are fast runners.

50. A. YES
B. NO
C. MAYBE

Here is a reminder of the meaning of the possible answers.

A. YES It must be true.
B. NO It can't be true.
C. MAYBE It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

---

51. Suppose you know that

All of the boys in the class collect stamps.
All students who are not members of the Stamp Club also do not collect stamps.

Then would this be true?

At least some of the boys in the class are not members of the Stamp Club.

51. A. YES
B. NO
C. MAYBE

---

52. Suppose you know that

All of the boys are running, but not everyone is running.

Then would this be true?

At least some of the people not running are not boys.

52. A. YES
B. NO
C. MAYBE

---

53. Suppose you know that

None of Tom's books are on the shelf.
No science books are on the shelf.

Then would this be true?

At least some of Tom's books are science books.

53. A. YES
B. NO
C. MAYBE

---

54. Suppose you know that

All of Bill's five uncles are allowed to drive.
All people who have a license have passed a driving test.
All people who are allowed to drive have a license.

Then would this be true?

At least one of Bill's uncles has not passed a driving test.

54. A. YES
B. NO
C. MAYBE

Here is a reminder of the meaning of the possible answers.

A. YES It must be true.
B. NO It can't be true.
C. MAYBE It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

---

55. Suppose you know that

All of the band members are working.

Then would this be true?

Everyone who is not working is also not in the band.

55. A. YES
B. NO
C. MAYBE

56. Suppose you know that

All the books on the shelf belong to the library.
No science books belong to the library.
At least some of the books that Elmer likes are on the shelf.

Then would this be true?

At least some of the books that Elmer likes are not science fiction.

56. A. YES
B. NO
C. MAYBE

57. Suppose you know that

All the people who live on Main Street were born in Milltown.
None of the students in Room 352 live on Main Street.

Then would this be true?

None of the students in Room 352 were born in Milltown.

57. A. YES
B. NO
C. MAYBE

58. Suppose you know that

At least some of Mr. Jones' students ride the bus to school.
All students who live on Route 55 own dogs.
All students who ride the bus to school live on Route 55.

Then would this be true?

None of Mr. Jones' students own dogs.

58. A. YES
B. NO
C. MAYBE

Here is a reminder of the meaning of the possible answers:

A. YES It must be true.
B. NO It can't be true.
C. MAYBE It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

---

59. Suppose you know that

All Y's are X's.
No Z's are Y's.

Then would this be true?

No Z's are X's.

59. A. YES
B. NO
C. MAYBE

60. Suppose you know that

All teachers are college graduates.
All people who have gone to high school are men.
All college graduates have gone to high school.

Then would this be true?

At least some teachers are not men.

60. A. YES
B. NO
C. MAYBE

61. Suppose you know that

All Z's are Y's.
No X's are Y's.
All T's are Z's.

Then would this be true?

No X's are T's.

61. A. YES
B. NO
C. MAYBE

62. Suppose you know that

All students who do not have a star are also not swimmers.
Frances is a swimmer.

Then would this be true?

Frances does not have a star.

62. A. YES
B. NO
C. MAYBE

Here is a reminder of the meaning of the possible answers.

A. YES It must be true.
B. NO It can't be true.
C. MAYBE It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

---

63. Suppose you know that

All the people in the auditorium are watching a movie.
All students in the senior play are in the auditorium.
Esther is a student in the senior play.

Then would this be true?

Esther is not watching a movie.

63. A. YES
B. NO
C. MAYBE

---

64. Suppose you know that

All birds have three eyes.
No ducks are birds.

Then would this be true?

No ducks have three eyes.

64. A. YES
B. NO
C. MAYBE

---

65. Suppose you know that

No Z's are Y's.
No X's are Y's.

Then would this be true?

At least some Z's are X's.

65. A. YES
B. NO
C. MAYBE

---

66. Suppose you know that

All of the red pencils are broken.
Emil's pencil is not broken.

Then would this be true?

Emil's pencil is not red.

66. A. YES
B. NO
C. MAYBE

Here is a reminder of the meaning of the possible answers.

A. YES It must be true.
B. NO It can't be true.
C. MAYBE It may be true or it may not be true. You weren't told enought to be certain whether it is "YES" or "NO".

---

67. Suppose you know that

All Z's are Y's.
All things that are not X's are also not Y's.

Then would this be true?

At least some Z's are not X's.

67. A. YES
B. NO
C. MAYBE

---

68. Suppose you know that

At least some of Mrs. Brown's flowers are not roses.
At least some of the flowers in the flower show are not roses.

Then would this be true?

At least some of Mrs. Brown's flowers are in the flower show.

68. A. YES
B. NO
C. MAYBE

---

69. Suppose you know that

All the pencils in the box are yellow.
None of the broken pencils are yellow.
All Dick's pencils are in the box.

Then would this be true?

None of the broken pencils are Dick's.

69. A. YES
B. NO
C. MAYBE

---

70. Suppose you know that

All the people who live near the lake can swim.
None of the students in Mr. Smith's class live near the lake.

Then would this be true?

At least some of the students in Mr. Smith's class cannot swim.

70. A. YES
B. NO
C. MAYBE

Here is a reminder of the meaning of the possible answers.

A. YES It must be true.
B. NO It can't be true.
C. MAYBE It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

---

71. Suppose you know that

None of the houses on Main Street are made of brick.
Allan's house is not made of brick.

Then would this be true?

Allan's house is on Main Street.

71. A. YES
B. NO
C. MAYBE

72. Suppose you know that

At least some of the boys in the class have bicycles.
All those who are not here also do not have bicycles.

Then would this be true?

No boys in the class are here.

72. A. YES
B. NO
C. MAYBE

73. Suppose you know that

All dogs are red.

Then would this be true?

All animals that are not red are also not dogs.

73. A. YES
B. NO
C. MAYBE

74. Suppose you know that

All Mr. Smith's cars have polished bumpers.
The red car does not have a polished bumper.
All the cars in the garage are Mr. Smith's.

Then would this be true?

The red car is not in the garage.

74. A. YES
B. NO
C. MAYBE

Here is a reminder of the meaning of the possible answers.

A.	YES	It must be true.
B.	NO	It can't be true.
C.	MAYBE	It may be true or it may not be true. You weren't told enough to be certain whether it is "YES" or "NO".

75. Suppose you know that

No ducks are birds.
Nothing with large feathers is a bird.

Then would this be true?

At least some ducks have large feathers.

75. A. YES
B. NO
C. MAYBE

76. Suppose you know that

All alligators are smart animals.
All animals that cannot sing are also not smart.

Then would this be true?

At least some alligators cannot sing.

76. A. YES
B. NO
C. MAYBE

77. Suppose you know that

All the students who live in the country have pets.
Barbara does not live in the country.

Then would this be true?

Barbara does not have a pet.

77. A. YES
B. NO
C. MAYBE

78. Suppose you know that

All X's are Y's.
All Z's are T's.
All Y's are Z's.

Then would this be true?

At least some X's are not T's.

78. A. YES
B. NO
C. MAYBE

END OF TEST. GO BACK AND CHECK YOUR ANSWERS.

Trial Answer Sheet for Lower Elementary Students

Name: ______________________________

1

	YES
	NO
	MAYBE
	SKIP

2

	YES
	NO
	MAYBE
	SKIP

3

	YES
	NO
	MAYBE
	SKIP

4

	YES
	NO
	MAYBE
	SKIP

5

	YES
	NO
	MAYBE
	SKIP

6

	YES
	NO
	MAYBE
	SKIP

7

	YES
	NO
	MAYBE
	SKIP

8

	YES
	NO
	MAYBE
	SKIP

9

	YES
	NO
	MAYBE
	SKIP

Cornell Deduction Test, Subtests on Ordinal and Propositional
Reasoning, Form A P 1 2 3 4

APPENDIX C. SAMPLES OF TEACHING MATERIALS.

Because of space limitations, it is possible to give only a small sample of the materials that were used in instruction. The following exercises indicate the level of instruction at the beginning and end of the instructional periods in upper secondary school, and also indicate the level of instruction which was attained in upper elementary.

These exercises also show the use of the Euler circle system, the use of symbols to represent classes and sentences, and the introduction of some technical vocabulary. Heavy reliance was placed upon the use of such exercises as these.

Class Reasoning Exercises Used in 10th and 12th Grades, Consisting of Three Used Early in the Instructional Period and One Used at the End..............................C-1

Conditional Reasoning Exercises Used in 11th Grade, Consisting of Three Used Early in the Instructional Period and One Used at the End......................................C-8

A Class Reasoning Exercise Used toward the End of the Instructional Period in 4th and 6th Grades..................................C-15

A Conditional Reasoning Exercise Used toward the End of the Instructional Period in 5th Grade...........................C-17

Class Reasoning Exercises Used in 10th and 12th Grades, Consisting of Three Used Early in the Instructional Period and One Used at the End.

Name ______________________

Exercise 1.

1. Define "set" ____

____

____

2. Listed below are five groups of three objects each. What "set" does each group belong to?

a. my pencil, your pencil, the teacher's pencil - ________________.

b. a cocker spaniel, a terrier, a bloodhound - ________________.

c. bread, milk, meat - ________________.

d. books, classes, filmstrips - ________________.

e. basketball, baseball, wrestling - ________________.

3. List five "sets" of things you encounter every day:

a. ______________________________

b. ______________________________

c. ______________________________

d. ______________________________

e. ______________________________

4. Redefine "set" - in your own words and without looking back.________

____

____

5. Define "element" ____

____

____

6. Listed below are five "sets". For each one, name three elements that belong to the set:

a. books - ______________, ______________, ______________

b. dogs - ______________, ______________, ______________

Exercise 1, continued

c. nations - ______________, __________________, ______________.

d. subjects taken in school - _________, __________, ___________

e. days of the week - _______________, _______________,__________

7. List five objects in this room which are elements of sets:

a. ______________________

b. ______________________

c. ______________________

d. ______________________

e. ______________________

8. Redefine "element" - in your own words and without looking back.

____

____

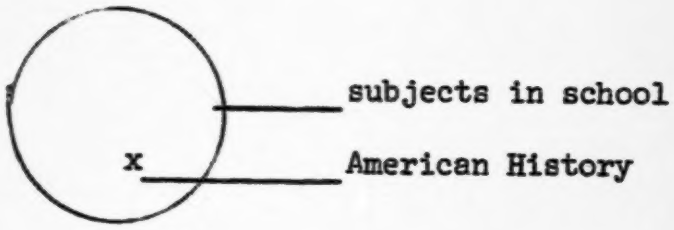
____

Exercise 2 Name ______________________

Each of the following pairs of words includes one set and one element of that set. In the space to the right of each pair draw a circle to represent the set, an x to represent the element, and label both.

I have done the first one for you so that you will understand what is to be done.

1. subjects in school
   American History



2. teachers
   Mr. Morrow

3. Monday
   days of the week

4. newscasters
   Chet Huntley

5. this trout
   fish

6. logicians
   Leonhard Euler

7. you
   students of logic

8. birds
   warm-blooded animals

Exercise 3. Name ____________________

1. Define "total inclusion" ____________________

____

____

2. Below are eight pairs of sets. Determine the relationship between them, and draw circle diagrams to represent that relationship.

Diagrams

a. universities

state universities

b. things to write with

pens

c. American nations

North American nations

d. summer months

months

e. suits

clothing

f. things which entertain

movies

g. states of the United States

southern states of the United States

h. presidential candidates

Republican presidential candidates

3. Redefine the relationship of total inclusion, using your own words and without looking back ____________________

____

Quiz Name ______________________________

Judge each of the following arguments. Decide whether the conclusion must follow - if it must, circle the work "VALID". If the conclusion either can't follow or may or may not follow, circle the word "INVALID".

1. At least some voters favor Wallace.

   All those who favor Wallace oppose President Johnson.

   Therefore, at least some voters oppose President Johnson.

   Valid Invalid

2. At least some dogs are carnivorous animals.

   Therefore, at least some carnivorous animals are dogs.

   Valid Invalid

3. At least some ducks are not wild.

   All wild animals are protected by law.

   Therefore, no ducks are protected by law.

   Valid Invalid

4. At least some Republicans do not support Rockefeller.

   All the Cayuga County supervisors are Repbulicans.

   Therefore, at least some Cayuga County supervisors do not support Rockefeller.

   Valid Invalid

5. At least some Assemblymen are not in favor of higher taxes.

   No one who approves civil rights is in favor of higher taxes.

   Therefore, some Assemblymen approve civil rights.

   Valid Invalid

6. At least some patrolmen use radar.

   All the men in this room use radar.

   Therefore, at least some of the men in this room are patrolmen.

   Valid Invalid

7. At least some Finns oppose Communism.

No Marxists oppose Communism.

Therefore, no Finns are Marxists.

Valid Invalid

8. At least some doctors are not surgeons.

All brain specialists are surgeons.

Therefore, at least some doctors are not brain specialists.

Valid Invalid

9. At least some motorcycles are not six-cylindered.

Therefore, at least some six-cylindered things are not motorcycles.

Valid Invalid

10. At least some cameras are expensive.

At least some Japanese products are not expensive.

Therefore, at least some cameras are not Japanese products.

Valid Invalid

11. At least some Cubans are not Communist.

At least some supporters of DeGaulle are not Communist.

Therefore, at least some supporters of DeGaulle are Cubans.

Valid Invalid

12. At least some Picasso paintings are valuable.

All valuable things are expensive.

Therefore, at least some Picasso paintings are expensive.

Valid Invalid

13. At least some magazines carry world news.

At least some daily publications carry world news.

Therefore, at least some magazines are daily publications.

Valid Invalid

14. At least some German scientists are in Berlin.

At least some pro-Westerners are not in Berlin.

Therefore, no German scientists are pro-Westerners.

Valid Invalid

15. At least some cancers can be cured.

Nothing that can be cured is a terminal case.

Therefore, at least some cancers are not terminal cases.

Valid Invalid

Conditional Reasoning Exercises Used in 11th Grade, Consisting of Three Used Early in the Instructional Period and One Used at the End.

'Exercise 1'

Find, label and put in parentheses the antecedents and consequents in each of the following:

1. If a dog is a beagle, then he can hunt rabbits.
2. If zombies sing, there is moonlight.
3. There are phlips if there are phlops.
4. Parking permits are legal if they are issued by the police chief.
5. If it requires real thought, homework is good.
6. If you should fail all your English tests, then you will definitely fail the course.
7. If conns are klobbered, klobs are conned.
8. I will go to the World's Fair if I can get a hotel reservation in New York.
9. That team of wrestlers must be good if they can beat the Ithaca team.
10. Cows recite poetry if the Queen of Hearts plays scrabble.

'Exercise 2'

Try making each of the following arguments valid by affirming the antecedent and concluding the consequent.

1. Premise - If (Joe makes that free throw), (we will win the game).
   Premise - ( )
   Conclusion - Therefore, ( )
2. Premise - If (there is a circle), then (there is a square)
   Premise - ( )
   Conclusion - Therefore, ( )
3. Premise #1 - (These are valuable books) if (they are hand-printed)
   Premise #2 - ( )
   Conclusion - Therefore, ( )
4. Premise - If (this specimen is an insect), then (it has three pairs of legs)
   Premise - ( )
   Conclusion - Therefore, ( )
5. Premise - (I will be late for school) if (I eat six eggs)
   Premise - ( )
   Conclusion - Therefore, ( )
6. Premise - If (John is a junior), (he is in Mr. Brown's homeroom)
   Premise - ( )
   Conclusion - Therefore, ( )
7. Premise - (The argument is valid) if (I affirm the antecedent)
   Premise - ( )
   Conclusion - Therefore, ( )
8. Premise - If (a whombit is a zabit), (a pringle is a plop)
   Premise - ( )
   Conclusion - Therefore, ( )

9. **Premise** - **(I will be unhappy) if (I don't win the contest)**

Premise - **( )**

Conclusion - **Therefore, ( )**

'Exercise 3'

Study each of these arguments, and indicate whether it is valid or invalid by circling the appropriate word.

1. If it is Saturday, then you can sleep late.
   It is Saturday.
   Therefore, you can sleep late. Valid Invalid
2. If you pass all your tests, you will pass the course.
   You have passed all your tests.
   Therefore, you have passed the course. Valid Invalid
3. If Mike is a dog, then he is an animal.
   Mike is an animal.
   Therefore, Mike is a dog. Valid Invalid
4. You must pay a fine if your library book is overdue.
   Your library book is overdue.
   Therefore, you must pay a fine Valid Invalid
5. If a car runs out of gas, it will stall.
   My car has stalled.
   Therefore, my car has run out of gas. Valid Invalid
6. Lemonade is bitter if there is no sugar in it.
   There is no sugar in this lemonade.
   Therefore, this lemonade is bitter. Valid Invalid
7. If we win the slalom, then we'll have a gold medal.
   We will win the slalom.
   Therefore, we won't have a gold medal. Valid Invalid
8. If p, q.
   p.
   Therefore, q. Valid Invalid

9. q if p

q

Therefore, p. Valid Invalid

10. We'll have a hootenanny if we can get the Rooftop Singers.

We can get the Rooftop singers.

Therefore, we'll have a hootenanny. Valid Invalid

'Exercise 14'

Name______________________________

Is the underlined conclusion valid?:

If a questionnaire is distributed to all adult members of a community, if this questionnaire asks whether the person thinks slave labor wrong, if there is at least 90% response, and if everyone tells the truth - then, the community really thinks slave labor is wrong, if at least 80% say they think slave labor is wrong. A questionnaire was distributed to all adult members of Smithtown, and it asked whether they thought slave labor was wrong. 95% of them responded to the questionnaire, and the people of Smithtown always tell the truth.

Now I know that Smithtown really thinks slave labor is right. Therefore, it is false that at least 80% of the adults of Smithtown said that they think slave labor is wrong.

p

q

r

s

t

u

Steps	Reasons:
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	

Steps **Reasons:**

10.

11.

12.

etc.

A Class Reasoning Exercise Used toward the End of the Instructional Period in 4th and 6th Grades

Grade____________________

Name____________________

Date____________________

Directions: Read the arguments carefully. Then using the Euler circle, diagram them. Circle the correct answer for each question. Remember, work against the conclusion but without breaking the rules. Good Thinking!

1. All Parisians are Frenchmen.
All Frenchmen are Europeans.
All Parisians are Europeans.

a) Valid
b) Invalid

2. No dogs are cows.
No cows are dogs.

a) Valid
b) Invalid

3. All Frenchmen are Europeans.
All Parisians are Frenchmen.
All Europeans are people.
All Parisians are people.

a) Valid
b) Invalid

4. All trout are fish.
All rainbows are trout.
No cats are fish.
No cats are Rainbows.

a) Valid
b) Invalid

5. All Parisians are Frenchmen.
No Ithacans are Parisians
No Ithacans are Frenchmen.

a) Valid
b) Invalid

6. No A's are B's. a) Valid
No B's are C's. b) Invalid
No A's are C's.

7. All lions are cats. a) Valid
All tigers are cats. b) Invalid
All lions are tigers.

8. All mice have tails. a) Valid
All rats have tails. b) Invalid
No mice are rats.

9. All A's are B's. a) Valid
No C's are A's. b) Invalid
No B's are C's.

10. All potatoes are vegetables. a) Valid
No Americans are vegetables. b) Invalid
No Americans are potatoes

A Conditional Reasoning Exercise Used toward the End of the Instructional Period in 5th Grade

Name____________________

Date____________________

Grade____________________

Directions: Using the method of writing the symbols right over the sentences, decide whether each of the following arguments is valid or invalid. Write valid or invalid in the space provided, whichever is correct. On the line below each argument, write what should be in the conclusion.

1.________ If this is an apple, then it grows on a tree.
This is an apple.
Therefore, it grows on a tree.

____

2.________ If this is a Buick, then it is an automobile.
It is not an automobile.
Therefore, it is a Buick.

____

3.________ If you like ice cream, then you will eat it often.
You do eat ice cream often.
Therefore, you like ice cream.

____

4.________ If this is a book, then it has pages.
This is not a book.
Therefore, it does not have pages.

____

5.________ If this is an inclined plane, then it is not a lever.
It is a lever.
Therefore, it is not an inclined plane.

____

6.________ If zebras have stripes, then so do tigers.
Zebras do not have stripes.
Therefore, nothing follows, necessarily.

____

7.________ It has a fulcrum, if it is a lever.
It is a lever.
Therefore, it has a fulcrum.

____

8.________ The man has a hammer, if he is a carpenter.
The man has a hammer.
Therefore, he is a carpenter.

____

9. ________ Elephants fly, only if birds have trunks.
Elephants do fly.
Therefore, birds have trunks.

____

10.________ If it rains tonight, we will catch a fish tomorrow.
If we catch a fish tomorrow, then we will eat it.
It rains tonight.
Therefore, we will eat it.

____

11.________ If we passed our grade, then we passed our subjects.
If we passed our subjects, then we studied.
We did not study.
Therefore, we passed our grade.

____

12.________ If wheee oooom, then gooosh braaack.

Wheee oooom.

Therefore, gooosh braaack.

____

13.________ Either you like sunny days or you like rainy days.

You like sunny days.

Therefore, you like rainy days.

____

14.________ If this is a seat, then you sit in it.

This is not a seat.

Therefore, you do not sit in it.

____

15.________ If girls grow beards, then, boys wear lipstick.

Boys do not wear lipstick.

Therefore, nothing follows, necessarily.

____

16.________ If %, then $.

Not $.

Therefore, not %.

____

17.________ If you are old enough, you may stay up until 9:00 P.M.

You may stay up until 9:00 P.M. only if you behave.

You are old enough.

Therefore, you behave.

____

18.________This is a simple machine, if it is a pulley.

This is not a simple machine.

Therefore, nothing follows necessarily.

____

19.________Sally's dress either has a belt or it is red.

Sally's dress does not have a belt.

Therefore, it is red.

____

20.________If hee, then ha.

Ha.

Therefore, hee.

____

21.________If this is a screw, then it has threads.

It does not have threads.

Therefore, nothing follows, necessarily.

____

22.________If Jane likes Betty, then she sent Betty a valentine.

Jane likes Betty.

Therefore, she sent Betty a valentine.

____

23.________If this has a fulcrum, then it is a lever.

It is a lever only if it is a simple machine.

It is not a simple machine.

Therefore, it does not have a fulcrum.

____

24.________If boo, then bee.

Bee.

Therefore, nothing follows, necessarily.

____

25._______This is a wedge only if it is not a pulley.

It is a wedge.

Therefore, nothing follows necessarily.

____

ERRORS

Page and Line No.

II-22, 1.21: H. P. Grice

III-3, 1.17: Change 'directly' to 'deliberately'.

III-3, 1.22: Quotes around "LNDT-2's"

III-8 & 9: Table No. III-2 instead of III-3

IV-10, 1.7: unless

IV-15: Replace line 3 and last two words of line 2 with the following: "...application of the concept when the operation has not been performed, in particular when other operations have been performed".

IV-27, 1.13: Omit comma after 'see'.

IV-28: Note #6 should read: "The correlations for CA for grades combined are based upon a random sample of LDT's. For conditional reasoning N=64; for class reasoning, N=82.

V-2, Piaget quote, 1.1: between

V-9, 1.19: Insert a comma after 'question'

V-12, 1.1: "...that arguments with suggestive content..."

VI-11, 1.9: Cronbach's

VI-24, end of line 25: Add 'be'.

VI-28, 1.17: inconsistent